EMPIRICAL MODELS OF INVESTMENT: A SURVEY

By
Demetris Ath, Georgoutsos,
University of Essex
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I. INTRODUCTION*

This paper attempts to provide a ctirical survey of the main models of investment that have been used in applied econometric work with particular emphasis paid on issues concerning their dynamic specification. It will be argued that the existence of convex costs of adjustment is a sufficient, although not a necessary condition for the dependence of the decisions of firms on future outcomes. To place this work in context it will be useful to provide a brief review[1] of the main tendencies in modelling the demand for factors of production and to look at some of the reasons that have given rise to their development.

We can discern two different modelling procedures in studying factor demand models, although recently they tend to get integrated into one. The first one is

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1. We do not intend here to provide a comprehensive survey of the literature. We rather focus on issues that we believe have primarily concerned researchers working in this area. For excellent surveys one can consult: Junankar (1972), Helliwell (1976), Nickell (1978) for investment, Killingsworth (1970), Hazledine (1981), Nickell (1986) for labour demand and Berndt (1981), Berndt et. al. (1981), Prucha and Nadiri (1986) for factor demand systems. Moreover the analysis that follows will be devoted to theoretical models suitable for empirical implementation and no attempt will be made to evaluate their empirical performance (see Clark (1979), Wisley and Johnson (1985), Bernanke et. al. (1988) for a comparison and evaluation of alternative investment models).

entirely static in its nature and its aim has been to study the structure of the production side of the economy or a sector of it. At the beginning it focussed on estimating the coefficients of a Cobb-Douglas or a Constant Elasticity of Substitution (CES) production function. The debate was about the returns to scale characterizing the production process and the value of the elasticity of substitution between capital and labour. The latter was considered to be important as it was related to issues like the effectiveness of investment incentives or the impact of wages rises on capital accumulation. The restrictive nature of the Cobb-Douglas and CES production functions led to the adoption of functions that placed no a priori restrictions on the Hicks-Allen elasticities of substitution and allowed for multiple inputs and outputs (see Fuss, McFadden and Mundlak (1978) for a survey). The benefits of this dissagregation are considerable since policy measures are often directed towards particular subsets of either the labour force (e.g. manual workers) or the capital stock (e.g. machinery). The costs of disaggregation are that the number of parameters to be estimated increase more rapidly than the number of inputs. This in turn makes the problem more difficult to estimate and the estimated parameters less reliable since results of the asymptotic econometric theory can not, as easily, be invoked any more. The increase in the number of parameters led to an effort to derive conditions for the separability of factors of production and consequently for the existence of consistent aggregation among them (see Berndt and Wood (1975), Denny and Fuss (1977), Berndt and Christensen (1973), Fuss (1977), Berndt and Wood (1979)). These conditions can be imposed by simply restricting the elasticities of substitution between various pairs of inputs to be equal. Moreover, the flexible production functions enabled researchers to test various hypotheses about the production function, like convexity of the isoquants, symmetry of input price effects, monotonicity et. cet.

In this approach, the first order conditions are derived by solving a static optimization problem of the representative firm. That involves either a profit maximization problem or a cost minimization one. These two alternative problems often produce quite different results (Burgess (1975)). Most of the researchers in this area, have chosen to work with the latter for two main reasons. First, under cost minimization, output is predetermined and no assumptions are required in order to specify the environment that the firm faces in its output market. Second, under profit maximization the derived demand function for each factor typically depends, in addition to the technological parameters and relative prices, on the levels of all the other endogeneous variables; in the case of a flexible production function the solution of the system of factor demands in terms of exogenous variables only (i.e. relative prices and, possibly, demand shift factors) poses a formidable problem. Thus, at the estimation stage, simultaneity biases will render the estimated parameters unreliable. In contrast to these problems, under cost minimization and after employing Shephard's Lemma we get the optimal levels of inputs as functions of the exogenously given relative prices and output only.

The second modelling procedure has been more preoccupied with the dynamic structure of factor demand models and it used to consist of the following two steps: first, derive the long-run equilibrium levels of the inputs of production from the optimization problem of the firm and second, employ a partial adjustment mechanism to model their gradual adjustment to the steady-state level.

We will present below the main models of investment behaviour and the approaches that have been adopted to the modelling procedures mentioned above. The neoclassical approach, presented in section II, was a first attempt to supply investment models with a rigorous microeconomic foundation although at the empirical level the estimated models did not bear a close resemblance to the theoretical ones. The cost of adjustment approach, discussed in section III, not only managed to provide a theory for the short run behaviour of investment but also succeeded in giving a rationale for the flexible accelerator models that had been so successful in empirical studies. Furthermore, it provided a theoretical basis for two of the most important developments in this area: first, interrelated factor demand models where disequilibrium in one factor demand market affects the demand for another; and second, Q-models which try to explain investment behaviour within a portfolio choice context (Keynes 1936, Tobin 1969). In the following section we present the putty - clay models and in the final part of the paper our results are presented.

II. THE NEOCLASSICAL APPROACH

The neoclassical paradigm, advanced by Jorgenson (1967), assumes that the firm aims to maximize the present value of its income. This policy, with the assumption of perfect capital markets and perfect certainty about the future, is consistent with that which maximizes the utility over a stream of consumption for those individuals who have claims on the income flows generated by the productive activity of the firm. Perfect capital markets allow a stream of income to be transformed to another provided that they both have the same present value. This implies that the firm acting in the interest of its shareholders should maximize the present value of its income irrespective of the preferences of its owners.

It is further assumed that the inputs of production can be transformed into output by employing a production process which can be represented easily by a well behaved production function. Furthermore, the capital stock is homogenous, i.e. equally productive, whatever its age and birthdate and it decays at an exponential rate. These are two crucial assumptions since the first allows us to aggregate capital stock of different ages whereas the second implies that capital decay is independent of the degree of utilization and the age structure of the capital stock. The firm faces no adjustment costs in the productive implementation of capital goods or in their disposal which implies that there exists a second-hand market for the homogenous capital goods (Nickell 1978). Finally, it is assumed that the firm is a price taker in the output and in the factors of production markets. Relaxing the asumption of perfect competition in the output market would allow for demand variables to affect the investment decision.

In the absence of any taxes the maximization problem of the firm can be written as:

$$\max_{\{L,\, I\}} \quad V = \int_{o}^{\infty} e^{-rt} \left[P_t Y_t - W_t L_t - v_t I_t\right] dt \tag{1}$$

$$\text{s.t.} \qquad Y_t = F\,[K_t,\, L_t] \tag{2}$$

$$\dot{K}_t = I_t - \delta\, K_t, \qquad K_o = \overline{K} \tag{3}$$

where V is the present value, as of time 0, of the firm, P_t, W_t, v_t are respectively the prices of output, labour and capital, K_t is the capital stock, δ is the rate of depreciation, and $\dot{K}_t$ its change per unit of time. We will finally assume, with no loss of generality, that the interest rate, r, is expected to remain constant. The first order conditions are the following:

$$\frac{\partial F_t}{\partial L_t} = \frac{W_t}{P_t} \tag{4}$$

$$\frac{\partial F_t}{\partial K_t} = \frac{v_t}{P_t}\left[r + \delta - \frac{\dot{v}_t}{v_t}\right] = \frac{C_t}{P_t} \tag{5}$$

where C_t is the user cost (or the "shadow" price of capital) i.e. the price required when hiring a unit of capital for one unit of time. Using equations (4) and (5) we can derive the optimal levels of capital and labour as functions of prices only i.e.

$$K_t^* = g\,(W_t/P_t,\, C_t/P_t) \tag{6}$$

$$L_t^* = h\,(W_t/P_t,\ C_t/P_t) \tag{7}$$

A few remarks are required on the above results. Firstly, despite the fact that the firm is involved in a dynamic optimization process it utilizes only current input prices to derive its optimal decision rules. The future time path of prices does not affect its capital stock decision. The same decisions would have been made if the firm was solving the instantaneous problem:

$$\max_{\{L,K\}} \ [P_t F\,(K_t\,,\ L_t) - W_t\, L_t - C_t\, K_t] \tag{8}$$

where C_t is the implicit price of capital defined in (5) above. The "myopic" decision rules (4) and (5)[2] are primarily a result of the assumption of no adjustment costs. If the firm can adjust its capital stock costlessly to the desired level each period there is no need for it to look into the future. Another reason for the "myopic" decision rules is that it has been assumed that there exists a second – hand market where the homogenous capital stock can be sold at the same price as the newly produced one after taking into account the amount that has been depreciated. If this assumption is relaxed and a lower bound of zero is introduced into the maximixation problem, it can be shown that the firm will have an incentive to form expectations about the future (see Nickell 1978, Arrow 1968). Consider, for example, the case of imperfect competition in the output market and assume that the firm expects a slump to occur in demand some time into the future. The optimal policy for the firm in this case will be to cease buying capital goods well before the slump begins. The firm will not like to be burdened with capital goods which are not going to be used and can not be disposed of either. If the investment decision is an irreversible process then the investment path of the firm will consist of spells of positive and zero investment.

Second, as we can see from condition (6) the neoclassical theory of investment provides us with a theory for the demand of the capital stock but not with one for investment. Substituting equation (6) into(3) and solving for investment we get that:

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2. A better interpretation of condition (5) comes from writing it in its equivalent form:

$$v_t = \int_t^{\infty} P_s F_{Ks}\, e^{-(r+\delta)(s-t)} ds.$$

The firm will go on investing up to the point where the cost of the machine will be equal to the discounted marginal revenue from the output of the machine.

$$I_t = g_{W/P}\,(\dot{W/P})_t + g_{C/P}\,(\dot{C/P})_t + \delta\, g\,(W_t/P_t,\, C_t/P_t) \qquad (9)^3$$

Equation (9) implies that investment is determinate and finite as far as the changes in prices are continuous. For discontinuous jumps in (C/P) or (W/P) we would need to appeal to adjustment costs to get an investment function (Haavelmo 1960)[4].

The models used by Jorgenson in his econometric investigation do not bear a close resemblance to the theoretical one presented above (e.g. Jorgenson 1965). What he actually does is to solve first for the optimal capital stock from condition (5). For a Cobb – Douglas production function we get that:

$$K_t^* = \frac{\alpha\, P_t\, Y_t}{C_t} \qquad (10)$$

i.e. the optimal capital stock depends positively on output and its price and negatively on the user cost of capital (parameter α denotes the share of capital in output produced). But note that if the firm is a price taker in the output market there is no justification for the presence of output in (10). Output is determined endogenously along with labour and capital and is a function of relative prices only[5]. The next step for Jorgenson was to assume that actual investment expenditure can be represented as a lag distributed function of the changes in the desired capital stock i.e.

$$I_t = \alpha\, \Sigma_{i=0}\, \lambda_i\, \Delta\, (PY/C)_{t-1} + \delta\, K_{t-1} \qquad (11)$$

Jorgenson relied on delivery lags of capital goods to justify the specification of equation (11). But this begs the question: if the firm is aware of the fact that it can not implement instantaneously its optimal decisions why does it not take this into account in its maximization problem? To claim that delivery lags are not expected but they are always occuring would clash with the assumption of

3. A term reflecting changes in demand factors could have been added if the assumption of imperfect competition in the output market had been adopted.

4. Jorgenosn's, (1967), answer to this problem is as follows: if a discrete change in the interest rate, for example, takes place there is going to be a compensating change in another component of the user cost of capital (e.g. $\dot{v}/v$) that will leave its current value unaltered. The future demand of capital will be affected of course from the new value of the interest rate.

5. One could appeal to a cost minimization problem to rationalize the presence of output. Alternatively a model of the rationed firm in the output market would give us similar results. However, in these cases we would not have any theory of the determinants of the firm's output.

perfect certainty. Another related remark is that since capital does not adjust within one period to its optimal level, output produced will not correspond to its optimal level either[6]. The inclusion of actual output, therefore, in equation (11) as a determinant of the desired level of the capital stock is wrong. Although the neoclassical model of investment can be, somewhat, improved with the inclusion of delivery lags its main weaknesses remain. The optimal capital stock will depend, in this case, on the outcomes of some fixed date into the future (assuming that there is a common delivery lag for all capital goods and for all the firms). Therefore, even in this case the net worth maximization problem boils down to one of maximizing net worth at each point in time. Moreover, what the model implies is that the firm will be aiming at a capital stock which it knows will not be optimal next period when expected changes in some exogenous variables will have taken place.

HI. COST OF ADJUSTMENT MODELS

As we have seen above, the main drawback of the neoclassical theory of investment has been its failure to provide a truly dynamic explanation of investment behaviour. We also noticed that one of the reasons for this was the assumption that the firm is always able to adjust instataneously to its optimal capacity. We could relax this assumption by introducing adjustment costs explicitly into our maximization problem: This implies that either output is lost because the existing production process is disrupted when adjustment takes place (costs internal to the firm) or that there are monopsonistic elements in the new capital goods' market which give rise to an upward sloping supply curve (costs external to the firm)[7]. However, a further assumption is required in order to get the lagged response of investment to changes in exogenous variables. The adjustment cost function must be strictly convex. Any other functional form would make it profitable for the firm to adjust instantaneously its existing capital stock to the

6. Assuming that there is a problem in adjusting employment too.

7. Keynes's (1936) derivation of the investment function (as presented by Witte (1963) was relying on an upward sloping supply curve for the newly produced capital goods. If the interest rates for example fell, an excess demand for capital goods would be created. This would cause the price of the new capital goods to rise, up to the point where the internal rate of return of the marginal unit of investment would be equal to the new level of the interest rates. Keynes provided a theory of the investment behaviour for the entire economy or a sector of it but not one for the individual firm. Finally, in the Keynesian model it were both the demand and the supply side of the capital goods market that determined the level of investment and not only demand as the case is in the neoclassical model (Junankar (1972), Mussa (1978), Precious (1987)).

optimal level and so we would be back to the behaviour implied by the neoclassical model. All the other assumptions of this last model are preserved in the present one too.

Deriving the first order conditions for the case where the adjustment cost function, C, is separable from the gross production function and it depends only on gross investment, C (I), it can be shown that;

$$v_t + C'(I)_t = \int_t^{\infty} e^{-(r+\delta)(s-t)} P_S \frac{\partial F}{\partial K} \{K_S, L_S\} \quad ds, \quad C' = \partial C/\partial I, \quad C'' > 0 \quad (12)$$

i.e. the firm will go on investing up to the point where the marginal cost of the additional unit of capital stock equals the present value of the additional revenues it generates. Condition (12) is not a decision rule since the future levels of the capital stock, on which the marginal product depends, are a function of today's investment. If we made the additional assumption of constant returns to scale and after having used the first – order condition for the flexible factor of production, i.e. labour, we can obtain (see Nickell (1978)):

$$C'(I_t) + v_t = \int_t^{\infty} e^{-(r+\delta)} P_S \frac{\partial F}{\partial (K_S/L_S)} \{W_S/P_S\} \quad ds. \qquad (13)$$

What equation (13) shows is that, when convex costs of adjustment are present, the investment decision of the firm at period t will depend on the entire future path of prices. This is the kind of result someone would expect to derive from a forward looking firm and this is in complete contrast to the "myopic" decision rules derived in the neoclassical model. It can be further shown that under static expectations the investment decision rule of the firm will follow the flexible accelerator[8] mechanism, that is:

$$\dot{K}_t = \gamma \, [K_t^* - K_t] \qquad (14)$$

where γ is the adjustment coefficient which is endogenous and K^* is the optimal capital stock derived from the stationary conditions. The adjustment coefficient is less than one, since it is the stable root of a second order difference equation, and it depends, among other things, inversely on the rate of interest. Therefore,

8. Concavity of the production function is also assumed. To derive the standard results of the static theory for the derived demand for factors of production, e.g. negative own price results, symmetry of cross price effects, strong separability must be imposed between the cost of adjustment and gross production functions (Treadway 1970).

the introduction of the costs of adjustment into the maximization problem has offered us with a rationalization of the flexible accelerator model which had been so popular in the early empirical studies of the investment behaviour[9]. If we relax the assumption of static expectations then it could be shown (Nickell (1978)) that the optimal level of gross investment can be written as:

$$I_t = \gamma \{K_t^* + \frac{\gamma + r}{1 + r} \Sigma_{s\, t+1}^{\infty} \left(\frac{1-\gamma}{1+r}\right)^{s-t} [K_s^* - K_t^*] - K_{t-1}\} + \delta\, K_{t-1} \tag{18}$$

where K_s^* is the desired capital stock which is a function of prices and factors affecting the position of the demand curve as of time s, and γ is the stable root of the difference equation obtained from the optimization problem. According to (18) the firm aims at a target capital stock which is a linear combination of the current period's desired level and an exponentially weighted sum of the difference between the former level and the desired capital stock for all future periods. Under static expectations these differences will be equal to zero and so we are back to expression (14).

There have been two interesting recent developments in the cost of adjustment models. The first extends the basic flexible accelerator model to incorporate the

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9. If one assumed that the desired level of the capital stock depended only on output, i.e. $K^* = uY$, equation (14) could have been written, in discrete form, as:

$$\Delta K_t = \gamma\, (uY_t - K_{t-1}) \tag{15}$$

Backward substitution would give us (after adding – up the replacement capital):

$$I_t = \gamma u\, \Sigma_{t=0}^{\infty} (1-\gamma)^i\, \Delta Y_{t-1} + \delta K_{t-1} \tag{16}$$

which says that gross investment is a weighted average of past changes in output. The derivation of equation (16) can be rationalized on the grounds that either the firm faces an irreversibility constraint on investment or that there exist costs in changing its capacity. In both cases the firm will want to know whether any change in current demand is going to be permanent or not. If intrepreneurs have adaptive expectations, a distributed lag of past levels of output is being used to approximate the notion of permanent income (Eisner 1967). In the flexible accelerator studies this last variable has been identified exclusively with output. $K^* = uY$ implies that either the technology is described in this framework by a fixed coefficient production function or that the relative prices have remained constant over the period under examination. As we can futher notice, at the empirical level, both the neoclassical and the flexible accelerator models end up with the estimation of almost identical equations (compare equations 11 and 16). The "naive" accelerator models were assuming that the firm can adjust costlessly and equations similar to (15) were employed. Another problem that arises with equations like (16) is that it is difficult to interpret the parameters (i.e. are they supposed to be the coefficients of the expectations formation mechanism or the parameters of the cost of adjustment technology?).

state of disequilibrium in the markets of the other quasi – fixed factors of production (Nadiri and Rosen 1969, Treadway 1970, 1971, 1974, Mortensen 1973). This implies that equation (15) should be written as:

$$\Delta K_t = \gamma_{k1} (K_t{}^* - K_{t-1}) + \Sigma_{j=2}^{n} \gamma_{kj} (X_{j,t}{}^* - X_{j,t-1}) \tag{19}$$

Expression (19) says that the demand for capital will depend not only on the present difference between planned and actual capital stock but also on the difference between planned and actual stock of factors X_j, $j=2, \ldots, n$. Therefore the first important feature of (19) is that it permits disequilibrium in other factors' markets to have spillover effects on the demand for another. The second interesting feature of (19) is that the difference between the short – run and the long – run elasticities of the jth factor will no longer depend only on γ_{jj} but on all γ_{ij}, $i \neq j$ [10]. This result indicates that short – run overshooting results can not be ruled out (Berndt, Morrison and Watkins 1981) [11, 12].

The second development in the costs of adjustment literature was the provision of microeconomic foundations for the Q theory of investment which has been one of the most popular approaches in investment theory after the publication of Tobin's article on "A General Approach to Monetary Theory" in 1969. Tobin's idea was to examine the investment decision problem in the more general context of portofolio choice. In this framework the rate of investment – the speed by which investors wish to increase the capital stock – is an increasing function of Q which is defined as the ratio of the market value of capital to its replacement cost [13]. The intuition behind this result is that if the market

10. Elasticities are defined with respect to any exogenous variable (i.e. relative prices and output) that affects $X_j{}^*$, $j=1, \ldots, n$.

11. A necessary condition for overshooting not to take place, is that the system of interrelated factor demand models has been derived as the solution to a dynamic optimization problem under convex costs of adjustment. In this case it can be shown that the characteristic roots of matrix M^* in the system $X_t - X_{t-1} = M_t{}^* (X_t{}^* - X_{t-1})$ will be inside the unit circle.

12. Nadiri and Rosen (1969) used a Cobb – Douglas production function and solved a cost minimization problem to derive functions for X^*, $j=1, \ldots, n$. Introducing a production function implies a certain number of restrictions between its coefficients and the adjustment coefficients b_{ij}, $i, j=1, \ldots, n$. The validity of the model can be tested by imposing the overidentifying restrictions (Faurot 1978).

13. In virtually all econometric work it is assumed that investment is demand determined – in accordance with the neoclassical and cost of adjustment theories of investment – and therefore the implicit assumption is made of sticky prices in the capital goods producing industry. It should be noticed that this is contrary to the Keynesian theory of investment where the demand and the supply price of capital goods are always equal.

valuation of a firm is greater than the replacement cost of its assets then it is more profitable to increase the capacity of the firm by buying up newly produced assets than by taking over an existing firm[14].

One of the great advantages of the Q – theory is that the market valuation of the firm incorporates all the expectations about the future stream of cash – flow generated from its assets and therefore it is not necessary to specify how expectations are formed. On the other hand there appear to be three drawbacks in this methodology: first, the stock market determines the value of firm's total assets but we are only interested on the value of fixed capital; second, we would like to know what the marginal value of capital is but what, at best, we can read from the stock market is the average value of it[15]; and third, share prices exhibit too much volatility[16] for one to believe that they reflect changes in the perception of entrepreneurs about the expected profitability of capital.

Abel (1979) and Hayashi (1982) have provided recently an interesting interpretation of the Q models of investment in the context of the cost of adjustment models. In the standard intertemporal problem of the firm under convex adjustment costs, marginal Q can be interpreted as being the shadow value of a unit of capital. As such, Q′ (Q′ = vQ) can be shown to be equal to:

$$Q'_t = \int_t^{\infty} P_s \, \partial \Pi_s / \partial K_s \, e^{-(r+\delta)(s-r)} ds \qquad (20)$$

where Π denotes the profit function. From (12) the first order conditions imply that:

$$v_t + C'(I_t) = \int_t^{\infty} P_s \, \partial \Pi_s / \partial K_s \, e^{-(r+\delta)(s-t)} ds = vQ_t, \quad C'' > 0 \qquad (21)$$

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14. This view is also shared by Keynes (1936), chapter 12, p. 151: "The daily revaluations of the Stock Exchange, though they are primarily made to facilitate transfers of old investments between one individual and another, inevitably exert a decisive influence on the rate of current investment. For there is no sense in building up a new enterprise at a cost greater than that at which a similar existing enterprise can be purchased; whilst there is an inducement to spend on a new project what may seem an extravagant sum, if it can be floated off on the Stock Exchange at an immediate profit".

15. There may be cases where the average and the marginal values of Q diverge substantially. For example after the oil shock in early 1970s the average value of Q for the excisting capital stock, which was energy intensive, fell below one while at the same time the incentive to invest in new, energy – saving, capital goods was high.

16. This has been one of the reasons why cost of adjustment coefficients, estimated from Q – models of investment, are very large compared to estimates derived from other methodologies. Since the Stock Market is much more volatile than investment one would expect to find the estimated adjustment cost coefficient to be high in order to rationalize the relatively slow adjustment of investment to changes in the Stock Market.

i.e. the firm will go on investing up to the point where the marginal cost of doing so is equal to the marginal benefit. Solving equation (21) for the rate of investment, I, we get that:

$$I_t = g\ (Q'_t - v_t) = g\ [v_t\ (Q_t - 1)] \tag{22}$$

where g is positive and depends on the parameters of the cost of adjustment function, C. Equation (22) provides a theoretical foundation to the Q theory where the rate of investment is an increasing function of the difference between the market valuation of an additional unit of capital, Q′, and its purchase price, v. After making the assumption of constant returns to scale in the net production function and using both the first order condition for the flexible factor of production and the transversality condition, Hayashi (1982) proved that marginal Q is equal to average Q. This was an important development since it gave a theoretical justification for the use of the measurable average value of Q in empirical studies of investment behaviour.

Recent research in Q-models has followed two directions. The first explicitly takes into account the financial policy of the firm and the system of corporate and personal taxation. The most interesting result derived is that the tax-adjusted value of Q is allowed to be different from one even at the steady-state situation. For example, if share repurchasing is not allowed the value of marginal Q will fall in the long run to the level determined by the ratio of the after personal - tax to the after - capital - gains - tax value of a pound paid to the investor. At this level shareholders will be indifferent between a pound distributed to them as dividend and one pound retained within the firm (Edwards and Keen 1984, Auerbach 1983). This result implies that when retained profits are the marginal source of investment finance the value of Q will be less than one, while, when new equity is issued Q will remain equal to one.

The second major, recent, development in the Q-theory has been to relax the assumption that there is only one quasi - fixed factor of production (therefore there is no need for capital to be treated as a homogenous good). Researchers, here, have tried to examine whether the results derived for the case of a single quasi-fixed factor of production can be extended to the case of multiple ones. This theoretical investigation has produced two main results. First, it is not in general possible to express total investment, in the case of many capital goods, as a monotonic function of Q. This can be attained only after imposing strict restrictions on the specification of the cost of adjustment function for each capital good (Wildansin 1984). Second, an investment function can be derived for each capital good separately, as a function of 1) a term incorporating the difference between the market value of the firm and the replacement cost of its entire capital stock, and 2) the flows and stocks of all the other capital goods. The higher the investment in another capital good, the greater the adjustment costs and, hence, the fewer resources that are available to be invested in the capital good we are interested in. On the other hand, the stocks of the other capital goods will have a positive effect since the higher they are the smaller the adjustment cost associated with their flows. Recent applications of these results can be found in Chirinko (1987) and Galleotti and Schiantarelli (1988) where debt and labour respectively are treated as quasi - fixed factors of production in addition to the capital stock.

IV. PUTTY-CLAY TECHNOLOGY AND INVESTMENT[17]

In both the neoclassical and the costs of adjustment models, examined above, it has been assumed that the firm can freely choose its optimal capital labour ratio from an infinite number of possibilities provided by a well behaved neoclassical production function (putty-putty technology). The technology available to the firm remains the same both before (ex ante) and after (ex post) an investment project has been undertaken. This implies that investment is always reversible (possibly with a cost), or in other words that the elasticity of substitution is the same ex post as ex ante. An alternative assumption to make, would be to adopt a putty-clay technology according to which a limit is set to the degree of flexibility in the substitution of factors of production upon the firm. In particular, the firm can choose ex ante from a whole range of technologies, provided by a neoclassical production function, to produce a given amount of output. However, once a decision has been made and the new capital goods have been installed, the amount of labour to be used with each unit of the new machines will be fixed for the entire life of the machine. According to this extreme version of the putty - clay models the ex post elasticity of substitution of capital for labour is zero.

The firm wishes to maximize the present discounted value of its net revenue stream from time 0 over an infinite horizon. The maximization problem can be expressed more formally as follows[18].

17. For theoretical and empirical studies using putty-clay models see: Ando et al. (1974), King (1972), Malcomson (1975), Malcomson and Prior (1979), Mizon (1974), Mizon and Nickell (1983), Nickell (1978) (1979), Schiantarelli (1983), Faini and Schiantarelli (1984), Mcintosh (1986).

18. We assume that firms have perfect certainty about the future evolution of prices and no costs of adjustment.

$$\max\ V = \int_0^{\infty} e^{-\rho t}\,[P_t Y_t - W_t L_t - v_t I_t]\,dt \qquad (23)$$

$$\text{s.t.}\ \ Y_t = \int_{-\infty}^{t} e^{-\delta(t-u)}\, I_u\, f[k_u]/k_u\, du \qquad (24)$$

$$L_t = \int_{-\infty}^{t} e^{-\delta(t-u)}\, I_u/k_u \qquad (25)$$

$$Y_t = z\,(P_t)\,\beta_t \qquad (26)$$

where V is the present value of the firm at time 0, and ρ is the discount rate. If we assume constant returns to scale, the output of the capital stock of vintage u, when it is first used in production, can be written as $I_u\, f[k_u]/k_u$, where k_u is the capital labour ratio employed by the technology of that vintage. This output is assumed to decline as the capital stock ages at an exponential rate δ[19]. Total output at time t is given by the sum of the output produced by each vintage at time t (equation 24). We further assume that no inventories are held and that demand is an inverse function of prices, P, and positively related to a shift factor β. Finally[40], demand for labour in given by the expression (25).

The first order conditions with respect to the capital labour ratio, investment and the optimal life of capital goods can be written as follows (see Malcomson 1975, Nickell 1979)[21]:

$$\int_t^{t+N_t} e^{-(\rho+\delta)(s-t)}\, M(P_s)\,[f(k_t) - k_t\, f'(k_t)]\, ds =$$

$$\int_t^{t+N_t} e^{-(\rho+\delta)(s-t)}\, W_s ds \;=\; \bar{W}_t \qquad (27)$$

$$\int_t^{t+N_t} e^{-(\rho+\delta)(s-t)}\, M(P_s)\,[f(k_t)/k_t]\, ds = v_t + \bar{W}_t/k_t \qquad (28)$$

$$M(P(t+N_t))\, f(k_t)/k_t\, e^{-\delta t} = W(t+N_t)/k_t\, e^{-\rho(t+N_t)-\delta N_t} \qquad (29)$$

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19. If we had allowed for embodied (or disembodied) technological progress an additional exponential term, say $e^{\lambda u}$ (or $e^{\lambda t}$), should have entered equation (24). In the latter case, output, of capital of vintage u, would be declining only if machine deterioration exceeded the rate of technological progress.

20. Above, we have assumed that capital goods decay exponentially and therefore that they are infinitely lived (physically). If their economic life is being determined endogenously and is finite, then the lower limit (i.e. minus infinity) should be replaced by $t-T$ where T is the age of the oldest capital good in use.

21. $M(P_t) = P_t\,(1 - 1/\varepsilon)$, ε = price elasticity of output.

The interpretation of these conditions is straightforward (we have assumed that the life of capital goods is finite and N_t denotes the lifetime of t vintage capital). Equation (27) tells us that by raising the labour - capital ratio on the t vintage by one unit (and keeping investment constant) we must generate enough revenues, over the lifetime of the vintage, to equate the costs associated with this change. The costs consist of the additional wages that the firm has to pay due to the more labour intensive technique built in vintage t. Similarly, equation (28), which gives the decision rule for investment in vintage t, implies that the marginal revenue from increasing the level of investment by one unit, but keeping the capital labour ratio intact, must be equal to the costs generated over the lifetime of the t vintage investment. Finally, equation (29) determines the optimal economic life of the t vintage capital. The optimal time for its scrapping is that at which if we increase the life of the machinery by one period we would generate just enough revenues to equate the costs from doing so. In other words, we require that the quasi fixed rent is zero at the marginal unit of the t vintage capital.

A simple manipulation of equations (27) and (28) would show that the capital output ratio is a function of W_t/v_t i.e. the firm has got to forecast the labour cost over the entire lifetime of the machine. Therefore, we have shown here that the putty-clay technology is sufficient to provide us with forward looking decision rules for the firm without having to rely on the more controversial convex costs of adjustment technology. Once the firm knows that its investment decision is going to be irreversible, there is no room for the "myopic" decision rules characterizing its behaviour in the neoclassical model.

It is straightforward to derive the investment equation in putty – clay models and show its dependence on future outcomes. If we differentiate equation (24) with respect to time t we derive the capacity accumulation equation which is given by:

$$\frac{\partial Y_t}{\partial t} = I_t \frac{f[k_t]}{k_t} - \delta Y_t \Rightarrow I_t = \frac{k_t}{f(k_t)} \left[\frac{\partial Y_t}{\partial t} + \delta Y_t\right] \qquad (30)$$ [22, 23]

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22. A great number of empirical studies on putty – clay models treat output as exogenous and work with the capacity accumulation equation (30). But of cource, output itself and its price are endogenous and depend on all the exogenous variables of the model (and therefore on $\bar{W}$).

23. Equations like (30) have been estimated empiricaliy, after assuming that there is a delivery lag and that the firm expects wages to rise at a given constant rate (Nickell 1978, 1979, Schiantarelli 1983). Moreover, an additional term usually appears in (30) which captures the echo – effect from the

Since k_t is a function of $(\bar{W}_t/v_t)$, the investment decision will be a forward looking one.

Summarizing our results up to now we must stress the following: first, unlike models employing the fixed coefficient production function (clay – clay models), in putty – clay models of investment relative prices matter; second, unlike the putty – putty (neoclassical) models where the capital stock can be reshaped next period if relative prices change, in putty – clay models the capital stock once installed is not malleable; third, within the putty – putty framework and given that depreciation is unrelated to use, it is never optimal for a profit maximizing firm to produce with idle capital since costs can be reduced by substituting labour for capital. In putty – clay models the possibility of production under idle capacity can not be ruled out because in this framework the ex – post elasticity of substitution between capital and labour is zero; fourth, an interesting testable implication of putty – clay models is that investment responds faster to changes in output than in relative prices. The intuition behind this last result is quite simple. Assume that there is an unanticipated change in either aggregate demand or in relative prices that imply the same increase in the long – run capital stock. In the case of the change in demand the firm can react immediately towards inccreasing its capital stock. On the other hand when relative prices change the firm must wait until old equipment becomes obsolescent (economically or physically) in order to invest in the new capital goods embodying the optimal labour – capital ratio. Bischoff (1969, 1971), among others, produced empirical evidence which confirmed this difference in the relative speeds of response of investment and claimed that these results support the putty – clay hypothesis. However, Abel (1981) has challenged this interpretation of the empirical results and has shown that the same pattern of responses can be derived from a dynamic model of investment based on a putty – putty production function and convex costs of adjustment[24].

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scrapping and therefore the required substitution of the oldest vintage of machines. In this case equation (30) is written as:

$$I_t = K_t/f(k_t)\,[\partial Y_t/\partial t + \delta Y_t] + e^{-\delta N^*} \frac{k_t/f(k_t)}{K^*_{t-N}/f(k^*_{t-N})} I_{t-N^*}$$

where N^* is the life of the oldest vintage of capital goods and $K^*_{t-N}/f(K^*_{t-N})$ its capital - output ratio.

24. Another hypothesis (not relying on costs of adjustment) to explain the discrepancy in the speed of response of investment within the framework of a putty – putty technology, could be to assume that expectations of relative prices respond slower to their own past values than the expectations of output do (Bischoff 1969, Abel 1981).

V. CONCLUSIONS

In this paper we have provided a brief review of the main models of the investment behaviour of the firm that have been used in empirical research. The original neoclassical approach which was considered to be static in its nature, has been replaced by the costs of adjustment one which offers a much better theoretical justification for the forward looking character of the investment decisions of the firms. Moreover this latter approach offers the necessary framework for the study of interrelated factor demand models and Q-models of investment. This last type of models has been the most popular one over the last fifteen years in both theoretical and applied work. The reason is that the share prices incorporate all the expectations that the investors hold about the future profitability of the companies. This implies that there is no need to model the expectations generating mechanism of the firms when information about the share prices is available.

The quantitave importance or even the existence of costs of adjustment in the investment decision of the firms has been seriously questioned by a number of researchers. However we have shown above that this assumption can be easily removed. We can derive very similar results when the much weaker assumptions of delivery lags or irreversibility in the investment decisions are being adopted.

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