MEASUREMENT AND DECOMPOSITION OF INEQUALITY BY POPULATION SUBGROUPS: A SURVEY AND AN EXAMPLE

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1. INTRODUCTION

As the term suggests, "inequality" can be viewed as a departure from an ideal case of "equality". Sen (1973, pp. 1-2) indicates that "the concepts of equity and justice have changed remarkably over history and, as the intolerance of stratification and differentiation has grown, the very concept of inequality has gone through radical transformation". Therefore, there exist a number of different interpretations of the meaning of equality and inequality. In everyday language inequality is associated with a notion of "difference" and "injustice"; it also has an emotive meaning, something like "unfairness". Nevertheless, for the purposes of the present study inequality is interpreted as any departure from the situation where each member of a population receives an equal share of what is to be distributed; let us assume it is income[1]. For convenience, it will be assumed that all distributions have the same mean and that all the population members have some positive income.

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1. Although, following the terminology of the theoretical literature in this area, in the theoretical part of this article we refer to the "distribution of income", the measurement and decomposition of inequality performed in the empirical part of the article are in terms of consumption expenditure per equivalent adult.

An index of income inequality can be defined as a "scalar representation of interpersonal differences in income within a given population"[2]. As Kanbur (1984) points out, there are two general approaches to the measurement of inequality; a positive and a normative. The first attempts to describe the pattern of income distribution and to summarize it in a single statistic. The second bases explicitly the measurement of inequality on value judgments related to the welfare lost due to the existence of inequality. As early as 1920, Dalton was arguing that underlying any index of inequality there is some concept of social welfare. Therefore, a comparison between the estimates of a particular index for two distributions involves an implicit or explicit normative judgment as to whether one distribution is to be preferred to another. Then, one can ask whether it is possible to rank unambiguously two distributions without using a specific index or inequality (and, hence, a specific Social Welfare Function). In order to answer this question, some diversion to the Lorenz curve is required. The Lorenz curve is defined as the relationship between the cumulative proportion of population members (arranged in ascending order of their incomes) and the cumulative proportion of their incomes. Hence, it is a convex function of the cumulative proportion of the population. In the case of perfect equality the Lorenz curve coincides with the 45° line and in the case of maximum inequality it coincides with the lower horizontal and the right vertical axis. Atkinson (1970) and Fields and Fei (1978) demonstrate that, if the Social Welfare Function underlying the inequality index is symmetric and equal to the sum of individual utility functions which, in turn, are increasing concave functions of the individual's income, a necessary and sufficient condition to rank two distributions without selecting a particular index is that their Lorenz curves do not intersect[3]. In this case, the distribution corresponding to the Lorenz curve closer to the line of perfect equality has a lower level of inequality. However, if the Lorenz curves of two distributions interect, different indices might give different rankings and, therefore, in order to rank them we should, first, select an index of inequality.

Although various authors have suggested different sets of desirable properties for inequality indices, there seems to exist a rather general agreement that an index should satisfy the following axioms:

Summerty axiom: Any permutation of incomes should leave the index unaffected.

2. Cowell (1977, p. 9).

3. Dasgupta et al (1973) show that this result holds even under the weaker assumption of S-concave utility functions.

Income - unit independence axiom: If the incomes of all population members change by the same proportion, the value of the index should remain unaffected.

Population - size independence axiom: If two or more identical populations are pooled, the value of the index should remain unaffected.

Transfer axiom: A regressive transfer of income between two population members which does not reserve their relative ranking should increase the index.

The fact that these axioms seem to be generally accepted does not imply that they are not controversial. Some authors suggest that the symmetry axiom may be undesirable because it does not take into account the process of income generation and the different circumstances faced by different population members[4]. The income unit independence axiom implies that the Social Welfare Function underlying the inequality index should be homogeneous of degree one with respect to the vector of incomes, which may be controversial. It has been suggested, instead, that the value of the index should remain unaffected if there are additions of equal amounts to all incomes[5]. However, if an index violates the income-unit independence axiom the degree of inequality depends on the unit of measurement of income, which is generally unacceptable[6]. The transfer axiom (which is also known in the literature as the *"principle of transfers"* or the *"Dalton - Pigou condition")* is considered by some authors as rather weak[7]. According to them the impact on the index of a regressive transfer of a given amount of income should be greater if the transfer takes place at a lower income level *("strong principle of transfers")*.* Doubts have also been expressed about the desirability of the population - size independence axiom[9]. A particularly desirable property for the purposes of the present work is additive decomposability. This property is discussed in the next section. In the rest of this section some of the most commonly used indices of inequality are presented, grouped into three categories: positive, entropy and normative.

4. See Sen (1979) and Cowell (1980). All the indices presented in this section satisfy this axiom.

5. See Dalton (1920), Kolm (1976a, 1976b).

6. Following the example of Kakwani (1980, p. 65), if we accept the "equal additions" instead of the "income-unit independence" rule, inequality can be diminished by, simply, calculating all incomes in cents instead of dollars.

7. See Sen (1973) and Kakwani (1980).

8. For stronger versions of the transfer axiom, assigning more weight to transfers at the lower than at the top end of the distribution, see Shorrocks and Foster (1987).

9. See Cowell (1977, pp 63-64). All the indices presented in this section satisfy this axiom.

1.1 Positive indices of inequality

These are indices of dispersion of incomes around a reference income level (usually the mean income of the population). Most of them are derived from statistical theory. As Kanbur (1984) points out, they are constructed in the following way. Firstly, a reference income level is selected. Secondly, the gap between the income of each population member and this reference level is calculated and weighted using an appropriate weighting system. Thirdly, the weighted gaps are summed and the mean weighted gap is calculated. Finally, this weighted gap is expressed as a fraction of the mean income.

Intutively, an obvious candidate to be used as index of inequality is the relative mean deviation

$$R = \Sigma_i |\mu - y_i| n\mu \qquad (1)$$

where y_i: the income of individual $(i = 1, \dots n)$
n: the size of the population
μ: the arithmetic mean income of the population $[\mu = \Sigma_i (y_i/n)]$

However, R is insensitive to transfers of income, as long as the persons involved are situated on the same side of the mean income and, hence, violates the transfer axiom; Several authors have suggested and used variants of R, which also violate the transfer axiom, as indices of inequality[10]. Another common statistical measure of dispersion of frequency distributions which can be used as index of inequality is, of course, the variance[11]

$$V = \Sigma_i (\mu - y_i)^2/n \qquad (2)$$

V has the appealing characteristic of attaching higher weights to larger gaps. As a result, a transfer of a given amount of income in the middle of the distribution has a much smaller impact on V relative to the transfer of the same amount at very high or very low income levels. However, as can be seen in (2), V depends on the mean income and, hence, violates the income-unit independence axiom. If V is divided by the mean income the squared coefficient of variation is derived, which does not violate the income-unit independence axiom[12].

10. See, for example, Kuznets (1957) and Elteto and Frigyes (1968). For a variant of R which satisfies the transfer axiom see Ebert (1988).

11. The square root of V (standard deviation) has also been used as index of inequality.

12. The square root of C has also been used as index of inequality.

$$C = \Sigma_i\ (\mu - y_i)^2/n\mu \tag{3}$$

Another popular index of inequality is the variance of the logarithms of incomes

$$L = \Sigma_i\ (\ln\mu^* - \ln y_i)^2/n \tag{4}$$

where μ^*: the geometric mean income of the population.

Since the expression inside the parenthesis in (4) can be written as $\ln(\mu^*/y_i)$, L satisfies the income-unit independence axion. However, Creedy (1977) demonstrates that a regressive transfer between two population members with incomes in excess of 2.72 times the mean income (in the case of natural logarithms) reduces the value of L instead of decreasing it. Therefore, L violates the transfer axiom. Nevertheless, Creedy also points out that the probability of a "violating transfer" is very low for most empirical distributions.

A common characteristic of R, V, C and L is the use of the mean income as reference income level. An alternative is to use each income in turn as reference level and calculate the mean of the resulting n^2 gaps as a fraction of the mean income. The resulting index is known as the relative mean difference ($j = 1, \ldots n$)

$$J = \Sigma_j\ \Sigma_i |y_i - y_j| / n^2\mu \tag{5}$$

Although (5) is similar to (1), it is easy to check that, unlike R, J satisfies the transfer axiom for any transfer of income. A summary measure of inequality closely related to J is the Gini index. This is undoubtedly the most well-known and widely used index of inequality. It is directly related to the Lorenz curve and can be defined as the ratio of the area between the Lorenz curve and the line of perfect equality over the area included between the lines of perfect equality and complete inequality. Several formulae for the Gini index have been suggested by different authors. The most well-known are the following[13].

$$G = \Sigma_j\ \Sigma_i |y_i - y_j| / 2n^2\mu \tag{6i}$$

$$G = 1 - \Sigma_j\ \Sigma_i \min\ (y_i, y_j)/n^2\mu \tag{6ii}$$

$$G = 1 + 1/n - 2[ny_1 + (n-1)\ y_2 + \ldots + 2y_{n-1} + y_n]/n^2\mu \tag{6iii}$$

where $y_i \leq y_2 \leq \ldots \leq y_n$.

13. For other formulae for the Gini index and rigorous treatment of its properties see Anand (1983, Appendix B).

Comparison of (5) and (6i) suggests that G is one half of J. G can be interpreted in a number of different ways. According to Sen (1973), if we take any pair-wise comparisons over the entire income distribution and assume that the person with the lower income suffers a depression (on finding his income to be lower) proportional to the income differential, then G is equal to the arithmetic mean of all such depressions in all possible pair-wise comparisons. Pyatt (1976) gives an interpretation of G which can be considered as the optimistic version of Sen's interpretation, within a game theoretic framework. He proposes a game in which each population member draws an income at random from the actual income distribution. If this income is higher than his own actual income he takes it, otherwise he retains his own. The mean expected gain of this game for the entire population expressed as a proportion of the mean income is equal to G. (6iii) implies that the Social Welfare Function underlying G is a weighted sum of the incomes of the population members. The weights are determined by the rank-order position of each member in the income scale. Consequently, the sensitivity of G to the transfer of a given amount of income does not depend on the size of the incomes of the two population members involved in the transfer, but on the number of population members between them in the income scale. Newbery (1970) demonstrates that if the individual utility functions are differentiable and strictly concave, then, there exists no additively separable Social Welfare Function ranking income distributions in the same order as G. Dasgupta et al (1973) show that the same result holds also for strictly quasi - concave utility functions. This fact makes G unacceptable if a utilitarian approach is adopted. However, as Sheshinski (1972) points out, additivity is a rather strong condition for a Social Welfare Function and if it is relaxed at least one Social Welfare Function ranking income distributions in the same order as G can be found[14].

In recent years several authors have attempted the construction of "ethically flexible" generalizations of G. These are indices based on the Lorenz curve, incorporating a "distributionally sensitive" parameter[15]. For example, Donaldson and Weymark (1980, 1983) present their class of "S – Ginis" which takes the form

$$G_S = (1/\mu)\ \{\mu - (1/n^{\delta})\ \Sigma_i\ [(n-i+1)^{\delta} - (n-i)^{\delta}]\ y_i\} \tag{7}$$

δ is the distributionally sensitive parameter ($\delta \geq 1$). The higher its value the more sensitive the index to changes at the lower end of the distribution. If $\delta = 1$,

14. Note, however, that the non – additive Social Welfare Function used by Sheshinski (1972) is quasiconcave but not strictly quasi – concave.

15. Therefore, these indices combine characteristics of both positive and normative indices of inequality.

$G_S = 0$ that is, the index is distributionally insensitive and, therefore, violates the transfer axiom. It is easy to show that for $\delta = 2$ G_S becomes the known Gini index. As δ *tends to infinity* G_S *tends to correspond to a* Rawlsian type of Social Welfare Function where the level of social welfare is determined exclusively by the level of income of the least well-off population member. G_S is usually presented as a Lorenz curve-based alternative to the Atkinson index of inequality which is presented below, but until now it has not been used extensively in empirical work[16].

1.2. Entropy indices of inequality

The concept of "entropy" was initially developed in information theory. It can be described, briefly, in the following way[17]. Assume that there are n independent events, each one with probability p_i $(0 \leq p_i \leq 1)$. When event i occurs, a number $h(p_i)$ is assigned to this information. If event i is likely $h(p_i)$ is low, that is $[dh(p)/dp] < 0$. In addition, since any two events are independent, the probability that both events i and j occur simultaneously is $p_i\, p_j$ and if it is further assumed that the information gain is additive, then $h(p_i\, p_j) = h(p_i) + h(p_j)$. The function that satisfies these properties is $h(p) = -\ln p$. Then, the individual information can be aggregated into a single number in order to calculate the average "information content" of the system using as weights the probabilities of the events. The resulting number $\Sigma_i\, p_i\, h(p_i) = -\Sigma_i\, p_i \ln p_i$ is known as the entropy of the system. Theil (1967) argues that the n events can be interpreted as the n population members and each probability p_i as the income share of member i, $y_i/n\mu$. Perfect equality ($y_i = \mu$ for each member) yields the maximum value of the entropy. Then, an index of inequality can be derived by subtracting the actual from the maximum entropy

$$
\begin{aligned}
T &= -\Sigma_i\, (1/n) \ln (1/n) + \Sigma_i\, (y_i/n\mu) \ln (y_i/n\mu) \\
&= -\ln (1/n) + (1/n)\, \Sigma_i\, (y_i/\mu) \ln (y_i/\mu) + (1/n)\, \Sigma_i\, (y_i/\mu) \ln (1/n) \\
&= \ln (1/n)[\Sigma_i\, (y_i/n\mu) - 1] + (1/n)\, \Sigma_i\, (y_i/\mu) \ln (y_i/\mu) \\
&= (1/n)\, \Sigma_i\, (y_i/\mu) \ln (y_i/\mu) \qquad (8)
\end{aligned}
$$

Theil (1967) also proposes another entropy index of inequality, in which the roles of population shares and income shares in expression (8) are reserved

16. See, also, the class of "extended Gini" indices suggested by Chakravarty (1988).

17. See Theil (1967), Cowell (1977) and Kanbur (1984).

$$N = \Sigma_i (1/n) \ln[(1/n)/(y_i/n\mu)]$$

$$= (1/n) \Sigma_i \ln (\mu/y_i) \quad (9)$$

Both T and N satisfy the axioms of symmetry, transfer, income-unit independence and population-size independence. However, a number of authors argue that they are very arbitrary and lack any intuition as indices of inequality. An additional disadvantage is that they do not have a constant upper bound. If one population member receives the total income, T takes the value of ln(n) whereas N cannot be calculated. In fact, even if a single population member has income close to zero, N tends to infinity.

1.3. Normative indices of inequality

The pioneering article on the construction of inequality indices explicitly based on Social Welfare Functions is that of Dalton (1920). According to the utilitarian approach used by Dalton, social welfare is the sum of individual utilities which are strictly concave functions of individual incomes (the same utility function for all individuals). As a result, the maximum level of social welfare is achieved when the -exogenously given- total income is equally distributed among all population members. Any departure from this situation reduces the level of social welfare. Dalton's index of inequality can, then, be defined as the difference between the maximum (potential) and the actual level of welfare over the maximum welfare level[18]

$$D = 1 - \Sigma_i U (y_i)/nU (\mu) \quad (10)$$

D has the disadvantage of not being invariant with respect to linear transformations of the utility function used in it. In order to avoid this problem, Atkinson (1970) introduces the concept of "equally distributed equivalent income per capita" (y_{EDE}), that is the level of income which if received by every population member would generate a level of social welfare equal to the level of social welfare generated by the actual distribution

$$nU (y_{EDE}) = \Sigma_i U (y_i) \quad (11)$$

Then, Atkinson's index of inequality can be defined as the difference between

18. Dalton himself suggested a slightly different formulation of this index.

the arithmetic mean income and the "equally distributed equivalent income per capita" over the arithmetic mean income.

$$A = 1 - y_{EDE}/\mu \tag{12}$$

If this index is to satisfy the income – unit independence axiom, the concave utility function U(y) must be limited the "constant elasticity of the marginal utility" form ($\varepsilon > 0$)

$$U(y) = \begin{cases} (y^{1-\varepsilon})/(1-\varepsilon) & \text{for } \varepsilon \neq 1 \\ \ln y & \text{for } \varepsilon = 1 \end{cases} \tag{13}$$

Therefore, the Social Welfare Function, $W(y) = \Sigma_i U(y_i)$, selected by Atkinson (1970) is homothetic, symmetric and additively separable to individual utilities and ε is the "inequality aversion parameter". The larger the ε the larger the weight attached to lower incomes. If it is equal to zero equal weights are attached to all individual incomes, whereas if it tends to infinity the Social Welfare Function tends to a Rawlsian type of Social Welfare Function. Combining (11) and (13), y_{EDE} ($\varepsilon \neq 1$) is given by

$$(y_{EDE})^{1-\varepsilon}/(1-\varepsilon) = [(1/n)\ \Sigma_i\ (y_i^{1-\varepsilon})]/(1-\varepsilon)$$

$$y_{EDE} = [(1/n)\ \Sigma_i\ (y_i^{1-\varepsilon})]^{1/(1-\varepsilon)} \tag{14}$$

and the Atkinson index of inequality is

$$A = 1 - (1/\mu)[(1/n)\ \Sigma_i\ (y_i^{1-\varepsilon})]^{1/(1-\varepsilon)} \tag{15}$$

if $\varepsilon = 0$, A violates the transfer axiom[19]. Otheriwise, A satisfies the axioms of symmetry, transfer, population – size independence and income – unit independence and has a straightforward interpretation. For example, if $A = 0.3$ 70% of the actual total income would be sufficient to generate the present level of social welfare, if it was equally distributed. Neverheless, A has been criticized on two grounds. Firstly, it has been argued that the utilitarian assumption of additive separability used by Atkinson (1970) for the construction of the Social Welfare Function underlying A is very strong and that non – individualistic, symmetric

19. If ε is greater than one and there are population members with zero incomes, social welfare tends to minus infinity, y_{EDE} cannot be defined ans A cannot be calculated. If $\varepsilon = 1$ A is equal to one minus the ratio of the geometric to the arithmetic mean income and if $\varepsilon = 2$ it is equal to one minus the ratio of the harmonic to the arithmetic mean income.

quasi - concave Social Welfare Functions can be used instead[20]. Secondly, Sen (1978) argues that the tasks of measuring inequality and the welfare loss due to the existence of inequality are completely different. However, the normative or "ethically flexible" inequality indices such as A, D and G_s implicitly confuse these two tasks[21].

2. INEQUALITY DECOMPOSITION[22]

In many studies judgments are made about the association of different factors with aggregate inequality. In recent year, a systematic attempt has been made to construct indices capable of decomposing aggregate inequality into its contributory components. In general, two types of inequality decomposition analysis can be distinguished. The first examines the contribution of inequality in the distribution of income from different sources to aggregate inequality *("inequality decomposition by factor components")*[2]*. The second examines the relationship between aggregate inequality and the levels of inequality of different population subgroups *("inequality decomposition by population subgroups")*[24]. This section presents the decomposition of three indices of inequality by population subgroups.

Decomposability of an inequality index means that if the population is grouped according to any external criterion into non - overlapping exhaustive groups, aggregate inequality can be decomposed into "between-groups" and "within-groups" inequality. The "between-groups" component of inequality can be defined as the value of the inequality index if every person in each group receives the mean income of that group (but the group mean incomes remain unchanged). The "within-groups" component is constructed from the population

20. See Sen (1973), Pyatt (1985).

21. "The idea of measuring inequality on the basis of an overall Social Welfare Function is fundamentally misconceived. It leads to a clear cut answer but to a question different from the one posed" [Sen (1978, p.92)].

22. This section draws on Anand (1983, Appendix C).

23. See Mangahas (1975), Fei, Ranis and Kuo (1978), Pyatt, Chen and Fei (1980), Shorrocks (1982).

24. For theoretical and empirical studies on the decomposition of inequality by population subgroups see Theil (1967), Fishlow (1972), Pyatt (1976), Bourguignon (1979), Fields (1979a), Shorrocks (1980, 1984), van Ginneken (1980), Blackorby, Donaldson and Auersperg (1981), Cowell and Kuga (1981), Das and Parikh (1982), Anand (1983), Mohan (1984), Cowell (1984, 1985), Adelman and Levy (1984, 1985), de Kruijk and van Leeuwen (1985), Glewwe (1986, 1988), Meager and Dixon (1987) and some unpublished works reported in Fields (1979b). See, also, Cowell (1980) who considers a class of decomposable indices which allow differential treatment of population subgroups.

share, the income share and the inequality index of each particular group, as an additively separable function over groups. Therefore, the contribution of each particular group to aggregate inequality can be identified. If the value of an index can be expressed as a weighted sum of the "within-groups" inequalities plus the "between-groups" inequality, the index is termed "weakly additively decomposable". Hence, if we have knowledge of changes in particular population groups, we can use a weakly additively decomposable index to evaluate their impact on aggregate inequality. The choice of different indices, inevitably, changes the relative importance of the "between-groups" and the "within-groups" components. Among the indices presented in the last section V, C, L, T and N, are weakly additive decomposable[25]. However, V violates the income-unit independence axiom and in the case of C the weights used for the construction of the "within-groups" component of inequality do not add up to unity and depend on the size of the "between-groups" component[26]. Therefore, it was decided to focus on the decomposition of T, N and L only (even though the latter violates the transfer axiom at very high income levels). The next subsections present the decomposition of these indices.

2i. The decomposition of Theil's T index

Assume that a population of n individuals belonging to K income classes can be assigned to J groups according to another variable (for example, region of residence or educational level). Then, the joint distribution of individuals by income and this variable can be given in the form of a matrix presenting the absolute frequencies n_{jk} of individuals in each cell (j, k) ($j=1, \ldots J$, $k=1, \ldots K$). Assume, also, that each individual in the kth income class receives the mean income y_k of that class[27]. Therefore, total income in cell (j, k) is $n_{jk}\, y_k$ and $\Sigma_j\, \Sigma_k\, n_{jk}\, y_k = \Sigma_j Y_j = Y$ is the total income of the population. $Y_j = \Sigma_k\, n_{jk}\, y_k$ is the total income of group j. Similarly, the total population size is given by

25. Although A is not weakly additived decomposable, some variants of A are; see Shorrocks (1980, p. 622).

26. The one–way decomposition of variance can be found in any standard statistical textbook; see for example Freud and Walpole (1980, ch. 15). For multivariate decomposition analysis of variance see Scheffe (1959, ch. 4). For the decomposition of the squared coefficient of variation see Theil (1967, p. 125).

27. This is not very restrictive because we can construct as many income classes as individuals. In fact, all the estimates of inequality indices presented below have been calculated from all the observations in the sample.

$n = \Sigma_j \Sigma_k n_{jk} = \Sigma_j n_j$ and $n_j = \Sigma_k n_{jk}$ is the population size of group j. The population and income shares of cell (j, k) are given by n_{jk}/n and y_{jk}/Y, respectively.

Extending (8), the first Theil index can be written as

$$T = \Sigma_j \Sigma_k (y_{jk}/Y) \ln [(y_{jk}/Y)/(n_{jk}/n)] \tag{16}$$

T can be decomposed into "between – groups" and "within – groups" components as follows

$$\begin{aligned} T &= \Sigma_j \Sigma_k (y_{jk}/Y) \ln [(y_{jk}/Y)/(n_{jk}/n)] \\ &= \Sigma_j (Y_j/Y)\ \Sigma_k (y_{jk}/Y_j)\ \{\ln[(y_{jk}/Y_j)/(n_{jk}/n_j)] + \ln[(Y_j/n_j)/(Y/n)]\} \\ &= \Sigma_j (Y_j/Y)\ \{(\Sigma_k (y_{jk}/Y_j) \ln[(y_{jk}/Y_j)/(n_{jk}/n_j)]\} \\ &\quad + \Sigma_j (Y_j/Y)\ \{\Sigma_k (y_{jk}/Y_j) \ln\ [(Y_j/n_j)/(Y/n)]\} \end{aligned} \tag{17}$$

The last term in (17) can be rewritten as $\Sigma_j(Y_j/Y) \ln[(Y_j/n_j)/(Y/n)]\ \Sigma_k(y_{jk}/Y_j)$, but since $\Sigma_k(y_{jk}/Y_j) = 1$ for each j, it is equal to $\Sigma_j(Y_j/Y) \ln[(Y_j/n_j)/(Y/n)]$ and (17) can be written as

$$T = \Sigma_j (Y_j/Y)\{\Sigma_k (y_{jk}/Y_j) \ln[(y_{jk}/Y_j)/(n_{jk}/n_j)]\} + \Sigma_j (Y_j/Y) \ln[(Y_j/n_j)/(Y/n)] \tag{18}$$

$\Sigma_k(y_{jk}/Y_j) \ln[(y_{jk}/Y_j)/(n_{jk}/n_j)]$ are the Theil indices T_j for each j and $\Sigma_j(Y_j/Y) \ln[(Y_j/n_j)/(Y/n)]$ is the value of T if every individual in j receives the arithmetic mean income of that group. Therefore

$$T = \Sigma_j (Y_j/Y) T_j + \Sigma_j (Y_j/Y) \ln[(Y_j/Y)/(n_j/n)] = T_W + T_B \tag{19}$$

where $\quad T_j = \Sigma_k (y_{jk}/Y_j) \ln[(y_{jk}/Y_j)/(n_{jk}/n_j)]$

$\qquad T_W = \Sigma_j (Y_j/Y) T_j$

and $\quad T_B = \Sigma_j (Y_j/Y) \ln[(Y_j/Y)(n_j/n)]$

T_W is the "within – groups" component of inequality, which is a weighted average of the group indices T_j, the weights being the income shares Y_j/Y of each group j. T_B is the "between – groups" component of inequality which is derived if the "within – groups" income differences are suppressed.

2ii. The decomposition of Theil's N index

As noted in section 1, N reverses the roles of the income share y_{jk}/Y and the population share n_{jk}/n in the formula of T. Therefore, keeping the notation unchanged and noting that for each j $\Sigma_k n_{jk}/n_j = 1$, N can be decomposed as follows

$$N = \Sigma_j \Sigma_k (n_{jk}/n) \ln[(n_{jk}/n)/(y_{jk}/Y)]$$

$$= \Sigma_j (n_j/n) \Sigma_k (n_{jk}/n_j) \{\ln[(n_{jk}/n_j)/(y_{jk}/Y_j)] + \ln[(Y/n)/(Y_j/n_j)]\}$$

$$= \Sigma_j (n_j/n) \{\Sigma_k (n_{jk}/n_j) \ln[(n_{jk}/n_j)/(y_{jk}/Y_j)]\} + \Sigma_j (n_j/n) \{\Sigma_k (n_{jk}/n_j) \ln[(Y/n)/(Y_j/n_j)]\}$$

$$= \Sigma_j (n_j/n) \{\Sigma_k (n_{jk}/n_j) \ln[(n_{jk}/n_j)/(y_{jk}/Y_j)]\} + \Sigma_j (n_j/n) \ln[(Y/n)/(Y_j/n_j)]$$

$$= \Sigma_j (n_j/n) N_j + \Sigma_j (n_j/n) \ln[(n_j/n)/(Y_j/Y)] = N_W + N_B \tag{20}$$

where $N_j = \Sigma_k (n_{jk}/n_j) \ln[(n_{jk}/n_j)/(y_{jk}/Y_j)]$

$N_W = \Sigma_j (n_j/n) N_j$

and $N_B = \Sigma_j (n_j/n) \ln[(n_j/n)/(Y_j/Y)]$

Naturally, since there exists a reversal in the roles of income and population shares between T and N, the weights in the "within – groups" component of inequality, N_W, are the population shares of the groups, n_j/n.

2iii. The decomposition of the variance of logarithms L

In order to proceed to the decomposition of L, further notation is required. Let $x_{jk} = \ln y_k$ (the same for all j); then, $x.. = (1/n)\Sigma_j \Sigma_k n_{jk} x_{jk}$ is the overall mean of x_{jk} and $x_{j.} = \Sigma_k n_{jk} x_{jk} / \Sigma_k n_{jk}$ is the mean of x_{jk} over k. Therefore, L can be decomposed in the following way

$$L = (1/n)\Sigma_j \Sigma_k n_{jk} (x_{jk} - x..)^2$$

$$= (1/n)\Sigma_j \Sigma_k n_{jk} [(x_{jk} - x_j.) + (x_j. - x..)]^2$$

$$= (1/n)\Sigma_j \Sigma_k n_{jk} [(x_{jk} - x_j.)^2 + (x_j. - x..)^2 + 2(x_{jk} - x_j.)(x_j. - x..)]$$

$$= \Sigma_j (n_j/n) \Sigma_k [(n_{jk}/n_j)(x_{jk} - x_j.)^2 + (1/n)\Sigma_j (x_j. - x..)^2 \Sigma_k n_{jk}$$
$$+ (2/n)\Sigma_j (x_j. - x..) \Sigma_k n_{jk}\ (x_{jk} - x_j.)] \tag{21}$$

Since $\Sigma_k n_{jk} = n_j$ and $\Sigma_k n_{jk}(x_{jk} - x_{j.}) = 0$, (21) can be expressed as

$$L = \Sigma_j(n_j/n)\Sigma_k[(n_{jk}/n_j)(x_{jk} - x_{j.})^2] + \Sigma_j(n_j/n)(x_{j.} - x_{..})^2$$
$$= \Sigma_j(n_j/n)L_j + \Sigma_j(n_j/n)(x_{j.} - x_{..})^2 = L_W + L_B \qquad (22)$$

where $L_j = \Sigma_k[(n_{jk}/n_j)(x_{jk} - x_{j.})^2]$

$L_W = \Sigma_j(n_j/n)L_j$

and $L_B = \Sigma_j(n_j/n)(x_{j.} - x_{..})^2$

Like N, the weights of the "within – groups" component of inequality in L are the population shares, n_j/n. In addition, since $x_{jk} = \ln y_k$, $x_{j.}$ is the logarithm of the geometric mean income of group j. Therefore, L is decomposable around the geometric (not the arithmetic) mean income.

Division of the "between – groups" ("within – groups") component by the total value of the index yields the "between – groups" ("within – groups") contribution to aggregate inequality. The higher the contribution of the "between – groups" component when the population is grouped by a particular variable, the stronger the association of that variable with aggregate inequality. It should be stressed that this is a mere statistical association which should not be interpreted as causality running from that variable to inequality (unless there is an underlying economic reasoning to support the idea of causality).

2iv. Stricity additively decomposable inequality indices

The class of strictly additively decomposable inequality indices is derived from the class of the weakly additively decomposable indices by changing the definition of the "within-groups" component. By symmetry to the definition of the "between-groups" component, the "within-groups" component is now defined as the value of the index if the group mean incomes are set equal to the overall mean income through an equiproportionate change in the income of every person within a group. In other words, the "between-groups" component is the value of the index for the hypothetical distribution where the "within-groups" inequality has been elimimated and vice versa. Let us examine whether the three indices considered in the previous subsections are strictly additively decomposable.

Taking into account that Yj = n^j and Y = ημ (where μ^ and μ are the mean incomes of group j and the entire population, respectively) (19) can be expressed as

$$T = \Sigma_j(n_j\mu_j/n\mu)T_j + \Sigma_j(n_j\mu_j/n\mu)\ \ln(\mu_j/\mu) = T_W + T_B \tag{23}$$

If all the "between-groups" inequalities are suppressed by setting all μ_j equal to μ, but the "within-groups" inequalities remain unchanged, T is equal to $\Sigma_j(\mu_j/\mu)T_j$, which is different from the "within-groups" component $(T_W = \Sigma_j(\eta_j/\eta\mu)T_j)$. Hence the elimination of "between-groups" inequalities reduces aggregate inequality by an amount different from T_B. $[T - \Sigma_j(\mu_j/\mu)T_j = T_B + (T_W - \Sigma_j(\mu_j/\mu)T_j) \neq T_B]$. Therefore, the "within-groups" component of inequality is not equal to the value of T when all the "between-groups" differences are eliminated and, as a result, T is not strictly additively decomposable.

Similarly, (20) can be written as

$$N = \Sigma_j(n_j/n)N_j + \Sigma_j(n_j/n)\ \ln(\mu_j/\mu) = N_W + N_B \tag{24}$$

Setting all μ_j equal to μ through equiproportionate changes in the income of every person within a group, so that "within-groups" inequalities do not change, N reduces to $\Sigma_j(n_j/n)N_j$ which is equal to the "within-groups" component of inequality, N_W. Hence, N is strictly additively decomposable.

Finally, if all the group geometric mean incomes in (22) are set equal to the overall geometric mean income (and, hence, all $x_{j\cdot}$ equal to $x_{\cdot\cdot}$), but "within-groups" inequalities remain intact, L is equal to $\Sigma_j(n_j/n)L_j$ which is, indeed, the "within-groups" component. Therefore, L is also strictly additively decomposable.

The fact that L is decomposable around the geometric instead of the arithmetic mean of the distribution and does not satisfy the transfer axiom over the entire range of incomes makes N the only inequality index which is strictly additively decomposable around the arithmetic mean and satisfies the four basic desirable axioms. For this reason Shorrocks (1980, p. 625) calls N "the most satisfactory of the decomposable measures". However, one may also require an index of inequality to have some intuitive justification and, as Fields (1979b, p. 424) points out "why...[N] should be used as a measure of economic inequality is far from transparent". It can be noticed that in both N and L the weights of the "within-groups" component are the group population shares, which are unaffected when "between-groups" inequality is eliminated. By contrast, the weights of the "within-groups" component of T are the income shares of these groups, which change after the equalization of the group mean incomes. The advantage of the strictly additively decomposable indices over the weakly additively decomposable indices can be illustrated by the following example. Consider the following question (a) By how much would inequality decline if regional inequalities were eliminated? and (b) How much less inequality would be observed if regional differences were the only source of variation in the distribution of income? Strictly additively decomposable indices give the same answer to both questions, whereas weakly additively decomposable indices do not. Hence, it can be argued that only those inequality indices additive in the strict sense give an unambiguous measurement of the contribution of any particular variable (grouping) to aggregate inequality.

3. AN EXAMPLE

These section gives an example of measurement and decomposition of inequality using some of the indices reported in section 1. Since each inequality index corresponds to a different Social Welfare Function and the selection of a particular Social Welfare Function depends on one's value judgments, it becomes evident that a single ideal index satisfying everybody's value judgments simply cannot exist. As a result, it was decided to use indices from all three groups mentioned above (positive, entropy and normative). More specifically, the following indices are used: the Gini index G, the Atkinson index A, the two Theil indices T and N and the variance of the logarithms L. In common with most empirical studies, the value of $\varepsilon = 2$ is used for the calculation of y_{EDE} in A[28]. The selected indices satisfy the axioms of symmetry, transfer, population - size independence and income-unit independence, apart from L which violates the transfer axiom at very high income levels. It is interesting to examine the type of transfers to which these indices are relatively more responsive. Using several hypothetical distributions, Champernowne (1974) demonstrates that A, N and L appear to be relatively more responsive to transfers at the bottom, G more responsive to transfers in the middle and T more responsive to transfers at the top of a distribution[29]. Hence, it can be argued that the combined use of G, A, T, N and

28. See Stern (1977).

29. In fact, Champernowne (1974) did not use these indices but some transformation of them.

L satisfies a wide range of tastes regarding the responsiveness of an index to different types of inequality.

The data used for the estimation of the above indices are the consumption expenditure microdata of the 1974 Household Expenditure Survey conducted by the National Statistical Service of Greece. In order to give an example of inequality decomposition, the population is divided into nine groups according to the region of residence and into two groups according to the size of the locality of residence. This particular grouping was selected because a number of authors on inequality in Greece [Geronymakis (1970), Prodromidis (1975), Voludakis and Panourgias (1980), Carantinos (1981)] and many politicians and policy - makers seem to suggest that a large part of the existing inequality in Greece emanates from disparities between regions and/or between urban and rural areas of the country. However, they rely on aggregate per capita data and-apart from Carantinos (1981)-do not substantiate their claim through decomposition analysis[30].

Taking into account that children and adults have different needs it was decided to use the distribution of consumption expenditure per equivalent adult for our estimation. Several adjustments were made to the original data before proceeding to the estimation of inequality indices. Firstly, expenditures on some lumpy items whose normalization period was considered to be longer than one year (purchases of cars and home repairs and improvements) were excluded from the definition of consumption expenditure. Secondly, 20 out of 7424 households were excluded from the sample on reliability grounds. Thirdly, since in 1974 the rate of inflation in Greece was relatively high, all the expenditures were expressed in constant average 1974 prices.

Equivalence scales for the cost of children were estimated using three different models (Engel-Rothbarth-Barten). Based on this empirical evidence, weights of 1.00, 0.40 and 0.25 were assigned to each adult, child aged 6-16 and child aged less than 6, respectively. Then, the total consumption expenditure of each household was divided by the number of equivalent adults in the household in order to obtain the consumption expenditure per equivalent adult of that household. The distribution of consumption expenditure per equivalent adult was derived by assigning the value of consumption expenditure per equivalent adult to each household member[31].

30. For references see Tsakloglou (1988).

31. Sampling problems, methods of adjustment and other technical problems are discussed in detail in Tsakloglou (1988).

The results of measurement and decomposition of inequality are presented in Table 1. Estimates of inequality indices for the entire population are reported in the bottom row of the table. In the first panel of the table the sample is split into nine groups according to the region of residence of the population member, Estimates of G, A, T, N and L are reported along with the mean expenditure and the population share of each group. The figures is parentheses under T, N and L are the percentage contributions of inequality "within" each region to aggregate inequality, according to the relevant index. These results suggest that in two regions (Thessaly and Epirus) inequality was higher than in the entire population. In addition, no clear relationship between inequality and mean regional expenditure can be observed.

Differences in regional mean expenditures appear to be quite substantial. The ratio of the mean expenditure per equivalent adult of the richest region (Greater Athens) over the relevant figure of the poorest region (East Macedonia and Thrace) was as high as 1.88. Therefore, at first sight, Geronymakis (1970), Prodromidis (1975) and Voludakis and Panourgias (1980) seem to be right in pointing out that there are serious disparities between the geographical regions of Greece. However, none of the decomposable indices gives a contribution of "between-regions" inequality to aggregate inequality higher than 14%. This result is important because it means that even if the government could redistribute consumption expenditure so that the mean consumption expenditure per equivalent adult for each region was equal to the national mean, but the level of inequality within each region remained unchanged (that is, if regional disparities were completely eliminated) aggregate inequality would not be reduced by more than 14%. In other words, in 1974 more than 85% of the existing inequality was due to the unequal distribution of consumption expenditure within the regions of Greece. Hence, our analysis contradicts the conclusions of the above authors.

Note also that for most regions the percetnage contributions of "within-regions" inequalities to aggregate inequality according to N and L are very similar and rather different from the perventage given by T. In addition, the higher the mean expenditure of a region the higher its "within - region" component of inequality according to T vis-a-vis its "within-region" component indicated by N and L. Taking into account, firstly, that T is relatively more sensitive to the existence of very high expenditures whilst N and L are relatively more sensitive to the existence of very low expenditures and, secondly, that the weights of the "within-groups" component of inequality are the expenditure shares in the case of T but the population shares in the case of L and N, these results are hardly surprising.

TABLE 1

Measurement and decomposition of inequality in Greece (1974)

Grouping factor	Populat. Share n_j	Mean Expend. of the group μ_j	Gini Index G	Atkinson Index ($\varepsilon = 2$) A	Theil Index T	Theil Index N	Variance of Logs L
REGION							
Greater Athens	0.317	4682	0.318	0.277	0.173 (35.3)	0.166 (26.9)	0.321 (26.3)
East Mainland and Islands	0.108	3729	0.314	0.285	0.164 (9.0)	0.165 (9.1)	0.335 (9.3)
Greater Salonica	0.073	3887	0.311	0.264	0.171 (6.7)	0.160 (6.0)	0.303 (5.7)
Central and West Macedonia	0.097	2859	0.311	0.280	0.160 (6.1)	0.162 (8.0)	0.329 (8.2)
Peloponnese and West Mainland	0.131	3269	0.318	0.285	0.180 (10.6)	0.165 (11.0)	0.334 (11.3)
Thessaly	0.098	2991	0.351	0.329	0.220 (8.8)	0.207 (10.3)	0.394 (9.9)
Crete	0.051	2914	0.328	0.294	0.190 (3.8)	0.179 (4.6)	0.341 (4.5)
Epirus	0.048	2811	0.343	0.326	0.206 (3.8)	0.199 (4.9)	0.389 (4.8)
East Macedonia and Thrace	0.078	2488	0.321	0.289	0.172 (4.6)	0.172 (6.8)	0.341 (6.7)
"Within – groups" component of inequality					0.177 (88.7)	0.172 (87.6)	0.337 (86.7)
"Between – groups" component of inequality					0.023 (11.3)	0.024 (12.4)	0.050 (13.3)
LOCALITY							
Urban (more than 10000)	0.568	4266	0.324	0.292	0.179 (59.5)	0.174 (50.4)	0.342 (50.2)
Rural (less than 10000)	0.432	2834	0.327	0.297	0.184 (30.9)	0.179 (39.5)	0.350 (39.1)

"Within – groups" component of inequality					0.181 (90.4)	0.176 (89.9)	0.346 (89.3)
"Between – groups" component of inequality					0.019 (9.6)	0.020 (10.1)	0.041 (10.7)
GREECE	1.000	3647	0.342	0.323	0.200	0.196	0.387

In the second panel of Table 1, the 1974 HES sample is split into two groups according to the size of municipality or commune of the individual's residence; urban (population more than 10000) and rural (population less than 10000). In 1974 the mean expenditure per equivalent adult in urban areas was more than 50% higher than in rural areas and inequality was higher in the rural than in the urban areas of the country. The latter of these results it is in line with the findings of Pashardes (1980), Carantinos (1981) and Athanasiou (1984), although our results indicate a far smaller inequality differential than the results of these authors[32]. This result (inequality being higher in rural than in urban areas) is rather unusual. Jain (1975) presents several (income) distributions for many countries for urban and rural areas separately and in most cases inequality appears to be higher in urban areas. A satisfactory explanation of why the evidence in Greece appears to be different might be the one offered by Pashardes (1980). He argues that part of the Greek high income (and, therefore, high expenditure) classes reside in suburban areas around big cities (Athens, Salonica). According to our classification these suburban areas have been included in the group of rural areas along with other agricultural municipalities or communes of similar or smaller size. This results in a bimobal distribution with high measures of inequality for rural areas.

The results of decomposition analysis show that only 9.6% (T), 10,1% (N) or 10.7% (L) of aggregate inequality could be attributed to differences between urban and rural areas. The results of the only other known attempt to decompose aggregate inequality in Greece [Carantinos (1981)1 are very different. Carantinos attempts a decomposition of aggregate inequality according to the dichotomy

32. According to Pashardes (1980) the Gini indices for the distribution of HHs by equivalent HH expenditure of the urban and rural areas in 1974 were 0.430 and 0.451, respectively. The relevant estimates of Carantinos (1981) for the distribution of HHs by total HH expenditure are 0.322 and 0.344. Athanasiou (1984) calculates the Gini index for the distribution of HHs by total HH expenditure to be 0.341 for the urban and 0.364 for the rural areas and the corresponding Gini indices for the distribution of individuals by per equivalent adult expenditure to be 0.270 and 0.287.

urban/rural areas according to Theil's T index, using the grouped consumption expenditure estimates for the distribution of households by total household expenditure of the 1974 Household Expenditure Survey. His results suggest that 40.7% of aggregate inequality was due to inequality within urban areas, 33,4% to inequality within rural areas and 25,9% to inequality between urban and rural areas. These estimates are strikingly different from the relevant estimates of Table 1. Part of the difference should be attributed to the differences in the data sets used. However, the difference in the contribution of the "between-groups" component should be attributed primarily to the fact that Carantinos uses a limited number of expenditure classes for his analysis. As noted earlier, the "between-groups" component of T is calculated using the group mean expenditures and the expenditure shares of the groups. Therefore, it is not affected by the fact that grouped data are used. However, the "within-groups" components are calculated using all the information available. Hence, the existence of some individuals with very high or very low expenditures within urban or rural areas increase the relevant T indices. If grouped data are used, these extreme expenditures affect only marginally the means of the relevant expenditures classes. Hence, in the study of Carantinos the estimates of T for urban and rural areas are downwards biased and the contribution of "between —groups" inequality is overstated[33].

The main finding of this section is that, contrary to the popular opinion, most of the observed inequalities in Greece are due to inequalities within regions and/or within urban and rural areas. Inequalities between regions and between urban and rural areas play a far less important role in the determination of aggregate inequality[34].

33. The use of grouped data gives relatively low estimates of the ratio of "within - groups" inequality over total inequality in the case of other studies, as well; see for example van Ginneken (1980).

34. In Tsakloglou (1988) it is shown that the distribution used in this section is approximately lognormally distributed and both "between-groups" components of inequality according to L, although not very high from an economic viewpoint, are highly statistically significant. In addition, when other factors are introduced into the analysis (multivariate decomposition of inequality), the direct contribution of regional inequality to aggregate inequality (main effect) drops to 2.3% and that of urban/rural disparities to 1.1%.

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