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BALANCING THE MEDIA BUDGET, REACH AND FREQUENCY IN MARKETING COMMUNICATIONS: A MACRO APPROACH

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ABSTRACT

A macro model for the balancing of the media budget, reach and frequency in marketing communications is presented. The model is solved for optimal policies under four distinct decision environment scenarios. The first scenario assumes that the media budget is given and the objective is to optimize the balance between reach and frequency. The second assumes that frequency is fixed and the objective is to select the optimal budget and the corresponding reach. In the third scenario it is assumed that management did not make prior commitments and the objective is, therefore, to optimize the media budget and the balance between reach and frequency. In the last scenario, the case of fixed reach with the objective of selecting the optimal budget and frequency levels is considered. Numerical examples are utilized throughout the discussion to demonstrate how the suggested model may be applied.

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INTRODUCTION

The goal of marketing communications is to influence the attitudes and/or purchase behavior of a targeted market segment. In advertising campaigns of a given media budget, two of the key factors that determine the degree to which this goal is attained are the media budget, the advertisements' reach and their frequency. Media budget is the sum allocated for the purchase of media space or time and does not include other non-media expenses.

Reach is the proportion of the targeted population that is exposed to a particular combination of media during a specific period [7, 20]. Defining reach as such, one refers to "effective" reach as the percent of target audiences reached at each effective level of advertising frequency [30; p. 34]. Finally, total reach is simply the number of people exposed to an advertising message at least once [2, 19]. In direct mail campaigns, reach is represented by a mailing list. In mass communications, reach is represented by the total unduplicated audience of all media employed.

Frequency refers to the average number of times the targeted audience is exposed to the same advertisement during a specific period [7, 20]. However, one refers to "effective" frequency when various degrees of advertising repetition are more (or less) effective in communicating a brand's advertisement or selling that brand [30; p. 331. In other words, vehicle exposure is a "necessary but not sufficient" condition for advertising exposure, and effective frequency should be evaluated in terms of advertising exposures or communication effects rather than vehicle exposures [13]. A follow up study of [13] found that, in practice, most media plan evaluations inflate estimates of effective reach. This implies that when media schedules are evaluated based on effective reach, media vehicle audience ratings should be weighted [16]. The study makes the distinction between vehicle and advertising audiences clearer. The assumption that one media vehicle exposure equals one advertising exposure, simply does not hold [30; p. 361. Thus, a media planner should know the number of vehicle exposures that equals the number of advertising exposures.

It has long been recognized that the media budget, reach and frequency interact in determining the effectiveness of an advertising campaign [1,2,4,6,8,12, 19,21,26]. Specifically, since an advertisement can influence only those exposed to it, its effectiveness is a function of its reach. Likewise, it is generally accepted that the degree to which an individual is influenced by an advertising campaign is a function of the number of times s/he is exposed to the advertisements. However, given a fixed advertising budget, the only way to increase frequency is by reducing the corresponding reach; and conversely, the only way to increase reach is by decreasing frequency. Thus, in order to optimize the effectiveness of a media plan, the proper balance between the media budget, the advertisement's reach and frequency of exposure must be determined.

Two distinct approaches may be taken in designing models for the communications problem outlined. First, one can take a micro approach. This approach considers each communication situation as unique, and accordingly, calls for the complete enumeration of all alternatives and their attributes; of all media, their costs and audiences in one case, and of all potential customers and the potential sales to each customer in the other [28, 31]. Some technique is then used in the determination of a particular communication plan. Second, one can take a macro approach. Under this alternative the overall relationships among key variables and the implications of these relationships are of interest [32].

As might be expected, the macro approach is applicable in the determination of long range plans and in the design of communication policies. The micro approach may be better suited for short term operational decisions.

Studies show, however, that there is a need for macro level models. A 1975 study [29], indicated that only four percent of interviewed advertisers had indicated the use of quantitative models in setting their advertising budgets. That was due to the difficulty in obtaining the needed input for existing models. Similar observations were made with regard to industrial advertisers [18]. The importance of applying quantitative models in advertising budgeting is illustrated by more recent studies [15, 23]. Specifically, [23] found that a significantly greater percentage (51%) of advertising managers were using more sophisticated techniques in comparison to San Augustine and Foley's results [29]. In addition, Lancaster and Stern, in a more recent study [15], showed that 75% of surveyed consumer advertisers use computers in their advertising budgeting decisions, although they suggest that some (77% of respondents) improvement, or much (4%) improvement is needed in advertising budgeting planning methods currently used. They suggest that budgeting programs should be more accessible and user - friendly [15].

The purpose of this paper is, therefore, to suggest a macro model for the communication problem outlined previously and to show how this model facilitates the formation of media policies and the setting of advertising media budgets.

ADVERTISING MEDIA MODELS

Early models for media selection, such as linear programming, did not distinguish between reach and frequency, but considered instead the total number of exposures [8, 9]. Such formulations assume that reaching, say, twenty individuals once is as effective as reaching four individuals five times. This assumption has been rejected on both theoretical and empirical grounds [7, 8]. Recognizing the distinction between reach and frequency, Roth [26] suggested that media should be scheduled so as to maximize gross rating point (GRP), subject to minimum levels of reach and frequency. Since GRP is computed by multiplying reach by frequency, it provides a measure of total exposure without drawing a distinction between its two components. It is known that the early LP models focused on measuring GRP's, although in a different context than the one utilized in this paper. GRP's are used by 89.4% of advertising agencies surveyed by Kreshel et al. [13]. Furthermore, this model does not suggest how the minimum levels of reach and frequency should be determined. Headen et al. [10] related TV advertising schedule variables to the attendant audience exposure pattern, developing a probabilistic model. The same researchers and Bearden [5], later used TV program ratings in a study of TV program exposure and attention. The study produced two equations for predicting attentive audience delivery of television advertising schedules. The study's important focus was on audience attention to TV programs instead of audience exposure to ads.

Advertising media models, applying to the one – media – vehicle situation, were developed early. Agostini (1962) developed reach estimation methods [3]. Specifically, estimated reach was defined as

$$R_N = R_{N-1} + (1 - R_{N-1})\ (a/Nb)$$

where N = insertions; a and b empirically derived coefficients [27; p. 8]. The use of two or more media vehicles may duplicate advertising efforts by creating an overlap in the audiences. The "Duplication of Viewing Law" developed by Goodhardt and Ehrenberg for the U.K., in 1969, was extended by Headen et al. [11] for the U.S. to include variables such as channel, program type, daypart, repeat viewing, and program ratings.

Frequency of exposure models were developed as well. Krugman [14] argued that three exposures to an ad are necessary in order for an individual to be effectively exposed to it. Krugman's and other researchers' models relied on the effective - reach criterion. This criterion, however, has a major drawback. Some individuals are exposed to the message an insufficient number of times and some are exposed to it an excessive number of times. As a result, allocation of advertising funds becomes ineffective. Extending the implications of this observation, one can conclude that the distribution of exposure frequency is an important variable for advertising managers to consider while budgeting the allocation of advertising expenditures.

A multivariate extension of the beta distribution is known as the Dirichlet distribution. The use of this distribution allows exposure to various vehicles to be different, so relacing the population homogeneity assumption. Rust and Leone applied the mixed-media Dirichlet multinomial distribution model on television and magazines [28]. Their analysis indicated that different media effectiveness, types of response, and degrees of interaction between the two components of this mixed-media schedule existed. Therefore, an integration of the media planning function, which will adequately model intermedia duplications between TV and magazine schedules, is suggested. Lackenby and Rice developed a network television exposure model which used single - insertion audience data as its only required input; the model is named the beta - binomial distribution - limited data model, BBD-LD [17]. That was found to produce the most accurate estimates of reach (average error 3.23%) and frequency (average error 18.77%) than both the beta binomial distribution indirect - estimation method and the univariate binomial model.

MEDIA SELECTION MODELS

The development of media selection models helps advertising managers select the optical media schedules subject to budget constraints and vehicles characteristics. Rust categorizes such models into three main categories: mathematical programming, simulation, and heuristics [27]. The most advanced of the media selection models employ advanced exposure estimation methods, efficient heuristic search routines, and easy to use decision support systems with advanced data base capabilities [27; p. 73]. These models call for a complete enumeration of all media to be considered and the attributes of each media in terms of its cost and audience. In these models, frequency is taken to be a variable which either directly or through intervening variables (e.g. through cumulative exposure level in the MEDIAC model [19]) influence the impact of advertisement inserts. A response function is, then, entered to indicate that the marginal impact of an advertisement decreases with the number of exposures [4, 19]. One should notice here that frequency is determined indirectly. Specifically, successive inserts in each medium are determined iteratively. A replication of an insert in a previously selected medium is treated as a new media option, albeit, with reduced effectiveness. Once the complete media budget is exhausted, the media schedule is complete and the average frequency can be computed [191.

The main advantage of the models outlined is their ability to provide a short term solution to the communication scheduling problem in a specific application situation. Their main disadvantage is the need to provide a complete enumeration of discrete alternatives; of all available media and their attributes in one case, and of all potential customers and their attributes in the other. The objective of the model presented below is to formulate the macro relationship among reach, frequency and the media budget as determinants of profitability and to derive their optimum levels. The formulation contained herein is not in the spirit of LP modeling, as that was previously discussed. For the sake of brevity the model is presented in terms of the mass media problem.

MODEL FORMULATION

This section presents a mathematical macro model, defines its functional components and suggests some functional forms for these components.

The objective of the model is to find the values of media budget, frequency and reach which maximize the expected net gain to be generated by the advertising campaign. Here, net gain is defined as total gross profits less the cost of advertising. It should be noted, however, that other measures of effectiveness, such as, total sales or recall, exist. In the opinion of the authors, net profit is the most applicable measure of effectiveness in a macro model.

Letting G be the expected net gain to be maximized, it can be maximized as a function of B – media budget, R — extent of reach, and F — frequency of exposure. Note that in this model, F is defined as a given frequency, or a frequency to be determined. The net gain is given by: $G = W[(MR)\ f_1\ (F) + M\ (1-R)\ f_1\ (0)] - B$. The model, in other words, addresses itself to a single population (e.g., a region, a nation, a city), without any consideration of variations over time (e.g., seasonality, or trend).

Beginning at the individual customer level, let W be the potential contribution to profit which the individual may contribute during the planning horizon. Let f_1 (F) be the proportion of W expected to be realized. It is assumed that f_1 (F) is a function of F, the number of times the individual is exposed to the advertisements. Thus, the conditional expected contribution to profit of an individual, given that s/he is exposed to the advertisements, is f_1 (F) W.

Turning from the individual customer to the total market, R is the proportion of the targeted population reached by the advertisements. Accordingly, the expected contribution to profits of a customer who is reached, is f_1 (F) W. Since not all potential customers are exposed to the advertisements, the expected contribution to profits of those not exposed is not ignored. This contribution can be easily shown to be $f_1\ (0)(1-R)\ W$ where f_1 (0) is the probability of sales to those not exposed.

After rearrangement of terms, the above relationship can be summarized in the net gain equation to be maximized:

$$G = [f_1 (F) R + f_1 (0)(1 - R)] \; WM - B \qquad (1)$$

where M is the market size and all other variables are as defined above.

To this point we have defined the gain equation as a function of three variables, i.e., F, B and R. We now show how these variables are functionally related. This is done for two reasons. First, it facilitates an analysis of the interaction between the variables in determining the net gain. Second, equation (1) can be simplified by the elimination of one variable. Since, by definition, F has to be an integer, and since for a macro model control over the budget is of prime interest, we find it convenient to eliminate R.

As noted earlier, given a fixed budget, reach and frequency are inversely related. We assume, therefore, that the budget available for exposing the advertisements once to all those reached is $B = zMRF$, thus $B = zMR$ for $F = 1$. It is recognized that this relationship may be difficult to apply in a macro model, where frequency may be affected by audience duplications, in which case frequency has to be interpreted as an average. In a macro model, however, where particular media options are not considered, B/F may be considered an acceptable presentation of the budget available per exposure.

It is generally accepted that reach is an increasing function of budget allocated for media purchase. Accordingly, let $R = f_2 (B,F)$ be a function of reach per replication of exposure. Substituting $f_2 (B,F)$ for R in (1) we get the general bivariate gain equation:

$$G = [f_1 (F) \; f_2 (B,F) + f_1 (0) \; \{1 - f_2 (B,F)\}] \; WM - B \qquad (2)$$

The functional forms of $f_1 (.)$ and $f_2 (.,.)$ are next discussed.

There is ample evidence to indicate that the impact of advertisements shows diminishing return as the number of exposures increases [9, 20, 22, 25]. Following published models for media selection which consider frequency [4, 19, 20], we assume that $f_1 (.)$ is the modified exponential function,

$$f_1 (F) = D - Ee^{-aF} \qquad (3)$$

where D is the saturation level of $f_1 (.)$. Here it represents the maximum share of a customer's potential contribution which might be realized.

$D - E$ is the value of $f_1 (0)$ and is the share of a customer's potential contribution to be realized when s/he is not exposed to the advertisements.

a is a parameter indicating how fast the function approaches its saturation level.

Parameters D and E, controlling the upper and lower limits of the function, are clearly dependent on the target market segments and the product line advertised. The last parameter, a, was assumed to be relatively stable around 0.66 for perceptual measures of advertising effectiveness, such as recall. Ray and Sawyer have shown, however, that this parameter may vary between 0.1 and higher values, depending on the measure of effectiveness employed [25]. It should be noted parenthetically, that Ray and Sawyer demonstrate how the parameters of f_1 (F) can be empirically estimated [24, 25]. There is, therefore, no further need to elaborate on this function. For illustrative purposes, the example utilized in this paper assumes the following values: $a=0.3$, $D=0.6$ and $E=0.54$.

As noted earlier, reach is an increasing function of the media budget alloted to each replication of exposure. It is generally accepted that above a certain level, this function shows diminishing returns [4, 26]. The diminishing returns result from advertising in less effective media in terms of the targeted market segments. This becomes self evident when the standard cost per thousand of a medium is adjusted to reflect the cost per thousand targeted audience. Before the point of diminishing returns, some authors assume that the function shows increasing returns due to the availability of efficient media which require substantial outlays [18, 26]. Recognizing that for a micro model a different function might be appropriate, f_2 (.,.) is assumed to follow an exponential function,

$$f_2\,(B,F) = 1 - e^{-cB/F} \tag{4}$$

where c is a parameter controlling the ascent of the function. This functional form was found adequate by an analysis of the mean cost of major media classes in relation to reach in particular markets. It was selected over more complex functional forms since there is conflicting evidence with regard to the existence of increasing returns [25]. To estimate c, the cost per thousand of major media classes is adjusted to the cost per thousand targeted segments. Given a number of such estimates, c can be estimated by an appropriate curve fitting technique. For illustrative purposes, in this paper, $c=4\times10^{-6}$ was assumed to give the best fit for f_2 (.,.).

MODEL OPTIMIZATION

The conceptual model developed in the previous section can now be restated as the following mathematical problem:
Maximize G where

$$G = [f_1\ (F)\ f_2\ (B,F) + f_1\ (0)\ \{1 - f_2\ (B,F)\}]\ WM - B \tag{5}$$

with F a positive integer. Substituting (3) and (4) into (5), we are faced with the task of optimizing G where G is given by

$$G = [(D - Ee^{-aF})\ (1 - e^{-cB/F}) + (D - E)\ e^{-cB/F}]\ WM - B \tag{6}$$

Four scenarios will be presented for this model. Each is presented below. Note that in any of them, once an optimal solution is found, reach can be found by solving (4).

Scenario 1: Fixed budget; find optimal frequency

In this scenario we will assume that the frequency F, is a continuous variable, so that the optimal value of the neighborhood of $F^*|B$ can be determined. After this is done, integral values of frequency in this neighborhood are evaluated to determine the true optimal value (integral). After the optimal value of F given B, noted $F^*|B$ is obtained, the evaluation of the integral values in the neighborhood of $F^*|B$ follows. The derivative of G with respect to F, noted G′(F) is

$$G'(F) = EWM\ [a\ e^{-aF} - (cB/F^2)\ e^{-cB/F} - \{a - (cB/F^2)\}\ e^{-aF - cB/F}] \tag{7}$$

Setting $G'(F) = 0$ we note that $F = (cB/a)^{1/2}$ is a solution to equation (7). It may be noted that as F approaches infinity G′(F) approaches zero and in the limit G equals $(D - E)$. The second derivative of G noted G″(F) is given, after some simplification, by:

$$\begin{aligned} G''(F) = EWM\ [&-a^2\ e^{-aF} - (cB/F^2)^2\ e^{-cB/F} \\ &- (2cB/F^3)\ e^{-cB/F}\ (e^{-aF} - 1) \\ &- (a - cB/F^2)^2\ e^{-aF - cB/F}] \end{aligned} \tag{8}$$

Evaluating at the point of interest, viz. $F = (cB/a)^{1/2}$ we obtain after some simplification (and letting $(aBc)^{1/2} = q$),

$$G''\left((cB/a)^{1/2}\right) = 2a^2\,(aBc)^{-1/2}\,EWM\,e^{-q}\,[1 - q - e^{-q}]\,[1 - (aBc)^{1/2} - e^{-q}] \tag{9}$$

which can easily be seen to always be negative for all values of $(aBc)^{1/2}$ different from 1.

Thus the optimal value of $F^*|B$ equals $(cB/a)^{1/2}$ and is truly a local maximum. A graph showing several iso – curves of B for selected values of F indicates the general shape of the gain G; it is easy to note that G is unimodal (see Figure 1). Selecting that budget which yields the highest value of G for any given F from the second scenario, Figure 2 shows G as a function of F, given the corresponding optimal budget. Figure 2 can be viewed as an envelope tangent to the budget iso – curves presented in Figure 1. Again, here one can see that G reaches a local maximum with respect to F.

A numerical presentation should be of interest at this point, based on data provided in Table 1. For values of $D = 0.6$, $E = 0.54$, $a = 0.3$, $W = 1$, $c = 4 \times 10^{-6}$ and $M = 25 \times 10^6$ we also present values of budget B that would yield integer values of $F^*|B$, reach (from equation 4) and the optimal gain for some selected values (Table 1).

For a budget of \$ 1,200,000 the gain is presented for various frequencies and the corresponding reach. The optimal gain occurs when the frequency is four.

Scenario 2: Fixed frequency; find optimal budget

Herein we assume that F is fixed and that only B is subject to control. Firstly, we obtain the derivative of G with respect to B, noted $G'(B)$ which is obtained directly from (6),

$$G'(B) = -(c/F)\,e^{-cB/F}\,(U - S)\,WM - 1 \tag{10}$$

where $U = D - E$ and $S = D - E\,e^{-aF}$. Setting $G'(B) = 0$ and solving we obtain the optimal value of B given F, viz.

$$B^*|F = -(F/c)\,\ln(T) \tag{11}$$

where $T = F\left((S - U)\,WMc\right)^{-1}$.

The second derivative of G with respect to B is noted $G''(B)$ and is given by

$$G''(B) = (cEWM/F)\ e^{-cB/F}\ (e^{-aF} - 1) \tag{12}$$

which is clearly negative. Thus we have found the maximum. A graph showing several iso – curves of F for selected values of B indicates the general shape of the gain G; it is easily noted that G is unimodal (Figure 3).

Selecting the frequency which yields the highest value of G for selected budgets from the first scenario, Figure 4 shows G as a function of B, given the corresponding optimal F. Figure 4 can be viewed as an envelope tangent to the frequency iso – curves of Figure 3. Again, it is clear that G reaches a local maximum with respect to B.

A numerical presentation follows. For the (same) values of D, E, a, W, c and M we present the optimal values of budget for various frequencies and the associated reach and gain in Table 2. For a frequency of four the gain is presented for various budgets. The optimal value of budget occurs at $ 2,244,310.

Some comments regarding the conditional optimizations seem appropriate at this time. It should be noted that the conditional optimizations do not yield the same results under any circumstances. Thus when $F = 4$ is assumed as the driving force the value of $B = 2{,}244{,}310$ is obtained. When $B = 2{,}244{,}310$ is driving force the value of $F = 7.69$ is obtained. These results should not be suprising if one examines equations $F^*|B = (cB/a)^{1/2}$ and $B^*|F = -(F/c)\ \ln\ [F\ ((1 - e^{aF})\ (cEWM))^{-1}]$ which is equivalent to equation (11). On the other hand, when F is allowed to be a continuous variable, the maximax solutions derived by either approach are the same. These maximax solutions are the optimal points of Figures 2 and 4. In this scenario, it was shown that a globally optimal solution does exist.

When F is indeed a constant, the solution was shown to be given by $B^*|F = (F/c)\ [\ln\ \{(S - U)\ WMc\} - \ln F]$, which can be written as $B^*|F = F\ [K_1 \ln K_2 - K_3 \ln F]$, or $B^*|F = F[K_4 - K_3 \ln F]$, and for ease of notation, we let $B^*|F = \theta$. Now, when F is a r a n d o m v a r i a b l e, with mean and variance E (F) and V (F) respectively, the above solution is modified to recognize the random character of the Frequency. In other words, $E(\theta)$ and $V(\theta)$ should be found, where:

$$E(\theta) = K_4\ E\ (F) - K_3\ E\ [F\ (\ln F)] \tag{12a}$$

$$V(\theta) = K_4^2\ V\ (F) + K_3^2\ V\ [F\ (\ln F)] \tag{12b}$$

Let P (F) be the probability function of F. Thus the formulae for E (F lnF) and V (F lnF) are originally presented.

$$E\,(F \ln F) = \sum_{k=0}^{\infty} k \ln k \; P\,(F) \tag{12c}$$

and

$$E\,\{(F \ln F)^2\} = \sum_{k=0}^{\infty} (k \ln k)^2 \; P\,(F) \tag{12d}$$

$$V\,(F \ln F) = E\,\{(F \ln F)^2\} - E^2\,(F \ln F)$$

One can now note that when $F=0$ or 1, $(F \ln F)=0$, thus the lower limit of the summations in (c) and (d) changes from 0 to 2. If, for example, we assume that P (F) is Poisson distributed,

$$\sum_{k=2}^{\infty} k \ln k \; e^{-f} f^k/(k!) = fe^{-f} \sum_{t=1}^{\infty} \ln\,(t+1)\; T_t$$

where: $T_t = \prod_{j=1}^{t} (f/j) = (f/t)\; T_{t-1}$, and $T_1 = f$, and

$$\sum_{k=2}^{\infty} k^2\,(\ln k)^2\; e^{-f} f^k/(k!) = fe^{-f} \sum_{t=1}^{\infty} (t+1)\,(\ln\,(t+1))^2\; T_t$$

Note that the following scenarios, 3 and 4, provide examples of bivariate optimization, where one variable is continuous and the other is discrete.

Scenario 3: Budget and frequency unknown and subject to control

Herein both F and B are considered subject to control. The gain G is therefore considered a bivariate function. The easiest way to perform the bivariate optimization of this function is by means of conditional optimization. For the moment we will assume F continuous. We will obtain $F^*|B$ and insert that value into G thus reducing G to univariate function of B only. The optimal value of B is obtained from the univariate G (B) function. This unconditional optimal value of B is then used to obtain the optimal value of F given the optimal value of B; this is noted $F^*|B^*$ and thus we have the unconditional optimal values of B and F. We then remove the continuity assumption on F and choose to examine the integer values in the neighborhood of F^* previously found. When examining these integer values for G we calculate the associated values of B^*/F from equation (11) calculate the gain and choose the maximum of the two gains. The associated values of B and F for the maximum gain are the true optima as far as the necessary conditions are concerned. The values are checked for sufficiency by means of second order conditions although from the Figures it is apparent that a maximum is achieved. Once F is given, one can obtain optimal B, or B*, and R*, from which G can then be obtained (the maximized value of G).

The optimal value of F given B, is equal to $(cB/a)^{1/2}$. Using this for F and inserting it into the gain G (eq. 6), we obtain (given $q = (acB)^{1/2}$)

$$G = WM\ [D + E\ e^{-2q} - 2\ E\ e^{q}] \tag{13}$$

Differentiating (13) with respect to B, we obtain

$$G'(B) = -1 + WME\ (ac)^{1/2}\ [(e^{-q} - e^{-2q})\ B^{-1/2}] \tag{14}$$

setting (14) equal to zero allows one to solve for the optimal value of B numerically. We note the optimal value as B*.

For the (same) values of D, E, a, W, c and M, we obtain, from (14), the unconditional optimal budget B* = \$ 3,201,930. From the relationship $F^* = (cB^*/a)^{1/2}$ we obtain F* = 6.53394. We now present the results for the integral values of F in the neighborhood of F*, viz. 6 and 7 and also the above unconstrained values.

Frequency	Budget (\$)	Reach	Gain (\$)
6	3,024,810	0.867	8,243,650
6.53394	3,201,930	0.859	8,263,340
7	3,346,780	0.852	8,250,060

Thus the constrained (integer F) solution occurs when the frequency equals 7.

Ordinarily sufficiency conditions for this third scenario would have been done by comparing the second mixed partial derivative of G, which is given by

$$-cEWMF^{-1}\ e^{-cB/F}\ ((e^{-aF} - 1)\ F^{-1}\ (cB/F - 1) - a\ e^{-aF}) \tag{15}$$

and its square to the product of second pure partials given in equations (8) and (12). The algebraic complexities dictate a numerical approach. A finite approximation may be used with finite differences and substituting the following

$$(G\ (F + 2\Delta F) - 2G\ (F + \Delta F) + G\ (F))\ (\Delta F)^{-2} \tag{16}$$

for the second pure partial with respect to F, and

$$(G\ (B + 2\Delta B) - 2G\ (B + \Delta B) + G\ (B))\ (\Delta B)^{-2} \tag{17}$$

for the second pure partial with respect to B, and now noting G as a bivariate variable,

$$(G\,(F+\Delta F,\ B+\Delta B)-G\,(F+\Delta F, B)-G\,(F,\ B+\Delta B) + G\,(F, B))\,(\Delta B\Delta F)^{-1} \qquad (18)$$

for the mixed second partial, then use these numerically as we would use the partial derivatives above.

For the optimal unconstrained solution derived above, viz., $F^* = 6.53394$ and $B^* = 3{,}201{,}930$, we let $\Delta F = F^* \times 10^{-4}$, $\Delta B = B^* \times 10^{-4}$ and obtain the following

From (7), first pure partial with respect to $F = 0$

From (12), first pure partial with respect to $B = (1.95)\ 10^{-4}$

From (16), the second pure partial with respect to $F = -585{,}585$

From (17), the second pure partial with respect to $B = (-1.22)\ 10^{-6}$

From (18), the mixed second partial of B and $F = -0.29874$
which clearly show F*, B* to yield a bivariate maxima for G.

In addition, a direct numerical approach is possible in that one would calculate G (F*, B*) and then show that it was greater than all of the following G (F* − ΔF, B*), G (F*, B* − ΔB), G (F* + ΔF, B*) and G (F*, B* + ΔB). Using values of one percent, we obtain

Function	Gain
G (F*, B*)	$ 8,263,430
G (F* − ΔF, B*)	8,262,930
G (F*, B* − ΔB)	8,263,030
G (F* + ΔF, B*)	8,262,938
G (F*, B* + ΔB)	8,263,030

A review of the above provides clear evidence that G (F*, B*) is indeed the optimal value of the gain equation.

Scenario 4: Reach is fixed; find optimal budget and frequency

Let us say that $R = f_2\,(B,F)$; $F = f_3\,(R,B)$, and $B = f_4\,(R,F)$. Then,

$G = [f_1 (F) R + f_1 (0) (1-R)] WM - B$ becomes:

$G = [f_1 \{f_3 (R,B)\} R + f_1 (0) (1-R)] WM - B,$

or $G = [f_1 (F) R + f_1 (0) (1-R)] WM - f_4 (R, F)$ (19)

If R is fixed, then two "sub–" scenarios are to be analyzed; specifically, solving for F, and solving for B.

Solving for F

From (19) and given that $f_1 (F) = D - Ee^{-aF}$, and $B = zRFM$, it is implied that $G = [(D - Ee^{-aF}) R + (D - E) (1-R)] WM - zRFM$, thus $G_F' = [aERe^{-aF}] WM - zR$, and setting this equal to zero, $aERWMe^{-aF} - zMR = 0$; $e^{-aF} = (zRM)/(aERWM) = z/(aEW) = Z$. Therefore,

$F^* = -(1/a) \ln Z$ (20)

(20) implies that F is independent of reach, since R is a constant. It does not matter what the value of R is.

Solving for B

Since $B = zMRF$, $F = B/(zRM)$. In addition, $f_1 (F) = D - Ee^{-aF}$. Thus, G now becomes: $G = [(D - Ee^{-aB/zRM}) R + (D - E) (1-R)] WM - B$, and its first derivative $G_B' = (a)/(zRM)\ REWMe^{-aB/zR} - 1$, which is set equal to 0, and solving for B, one gets: $-(aB)/(zR) = \ln [z/(aEW)]$. Setting $z_2 = z/(aEW)$, the solution is a linear function of R:

$B^* = [(-zR)/a] \ln (z_2)$ (21)

DISCUSSION

The model presented in this paper provides a macro approach to the communication scheduling problem. As shown in the previous section, calculus can be utilized in providing optimal solutions under four decision environments. The first case assumes that management has already decided on a media budget to be expended. The objective of the model is then to determine the optimal balance between reach and frequency, given the budget. In the second case, management has already fixed the frequency of exposure and the objective is to determine the optimal budget and resulting reach. In the third case, management is open-minded with regard to budget, frequency and reach. In the fourth case, management had the objective of selecting the optimal budget and frequency, given a fixed level of reach.

It should be emphasized that only the third alternative provides an unconstrained optimal solution. The other three provide optimal solutions conditional upon prior, non-optimal, decisions. It is therefore recommended that the third scenario be followed whenever feasible.

The other three scenarios are provided in recognition of situations which often occur in practice. It is not uncommon to find a strict corporate - wide policy with regard to advertising budgets which is based on different logic than the one suggested; for example, setting advertising as a percentage of sales. Given such a policy, the first scenario provides guidelines with regard to targeted reach and frequency. It is also not uncommon to have an advertising agency requiring a certain level of frequency. This is often the case in the introduction of a new product, when it is felt that a minimum threshold level of exposure is needed for the creation of an initial impact. Under such conditions, the second scenario provides the optimal budget and the resulting reach - a similar case with Scenario 4.

The model presented is applicable at the early planning stages of an advertising campaign. It facilitates the setting of an advertising campaign. It facilitates the setting of a budget to be allocated later to particular media options. It also suggests the level of reach and frequency to be expected from the final media schedule. Details, such as audience duplications, and minimum budget allocations to media options, are not explicitly modeled. Since this is a macro model designed for policy rather than operational decisions, the omission of micro details is justified. It is, therefore, recommended that this model be used in conjunction with a micro model such as the one suggested by Little and Lodish [19] for media planning or by Lodish [20] for sales calls planning. If the reach and frequency of the resulted micro schedule widely diverge from those suggested by the macro model, one would suspect that either different response functions were employed to represent the effects of repetition, or that the reach equation, $f_2(.,.)$, does not adequately represent the attributes of the media options ultimately considered. In the first case, corrective action is obvious, since there is no reason to assume different functions at the macro and micro models. In the second case, one should check whether the media options selected come from media classes used in estimating the parameters of the reach equation in the macro model.

The particular functional forms employed for the effects of frequency, and the extent of reach should not obscure the generality of the approach. These functions have been suggested previously by others and were found appropriate for the empirical data considered by the authors. Similar optimizations may be derived for the gain equation with other functions. If the mathematics gets to be too difficult to handle, numerical methods may be employed.

TABLE 1

Budget ($´s)	Frequency (given budget)	Reach	Gain (in million $´s)
0	0	0.0	1.50
75,000	1	0.259	2.33
1,200,000	4	0.699	6.89
3,675,000	7	0.878	8.22
7,500,000	10	0.950	6.19
12,675,000	13	0.980	1.78
14,700,000	14	0.985	−0.10

Optimal Gain

Frecuency	Reach	Gain (in Mill. $)
1	0.992	3.77
3	0.798	6.69
4	0.699	6.89
5	0.617	6.77
7	0.496	6.18
10	0.381	5.19

TABLE 2

Frequency	Budget (given frequency)	Reach	Gain (millions $'s)
0	0	0.000	1.50
1	659,690	0.929	4.09
4	2,244,310	0.894	7.69
7	3,346,780	0.852	8.25
10	4,088,320	0.805	7.74
13	4,561,650	0.754	6.92
14	4,671,860	0.737	6.63

Optimal Value of Budget

Budget	Reach	Gain (mill. $)
0	0.000	1.50
500,000	0.394	4.71
1,000,000	0.632	6.46
1,500,000	0.777	7.33
2,000,000	0.865	7.67
2,244,310	0.894	7.68
2,250,000	0.895	7.68
2,500,000	0.918	7.66
3,000,000	0.950	7.46
3,500,000	0.970	7.15

FIGURE 1

Net Gain as a Function of Frequency, Given the Budget

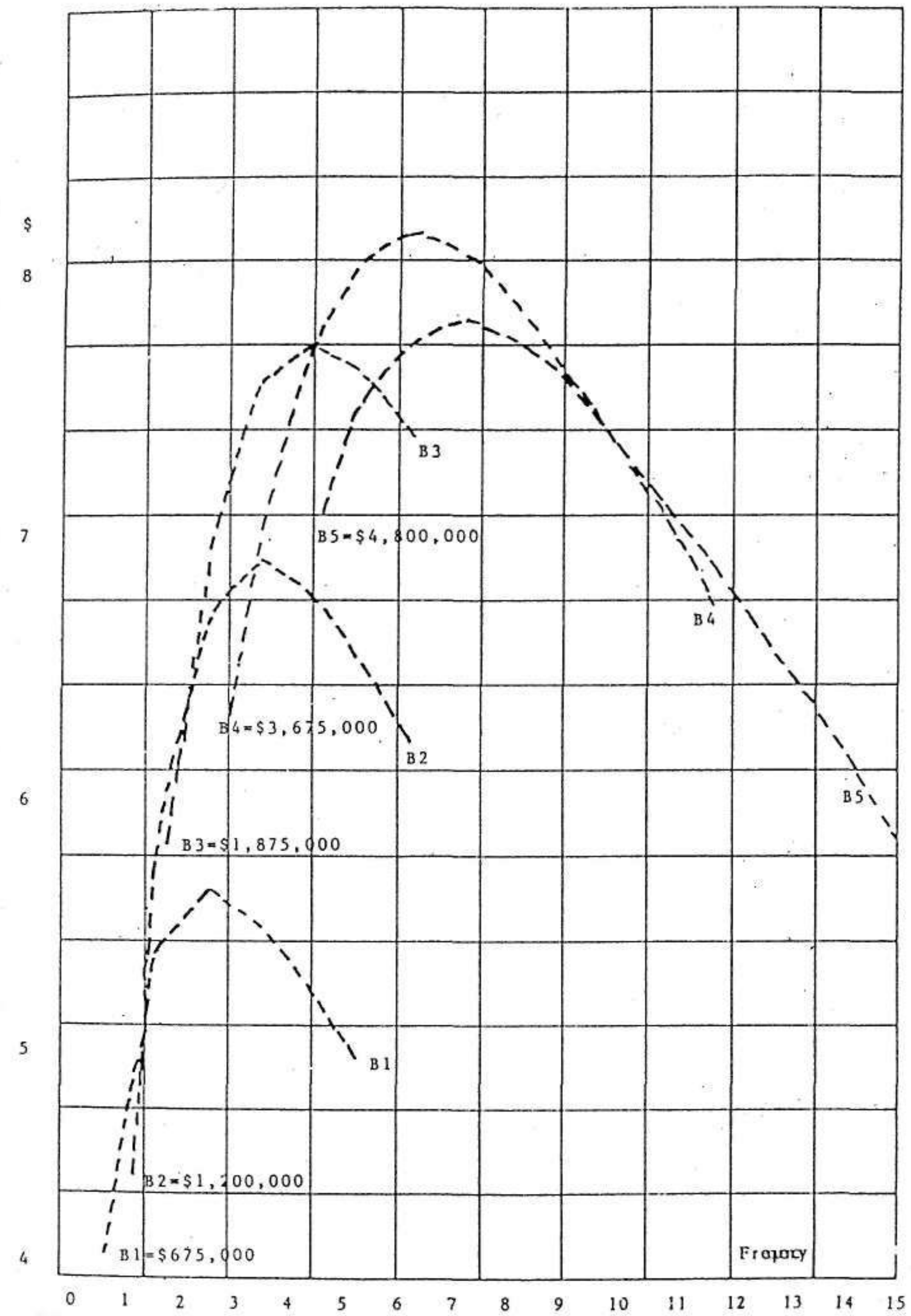


FIGURE 2

Net Gain as a Function of Frequency Given the Corresponding Optimal Budget

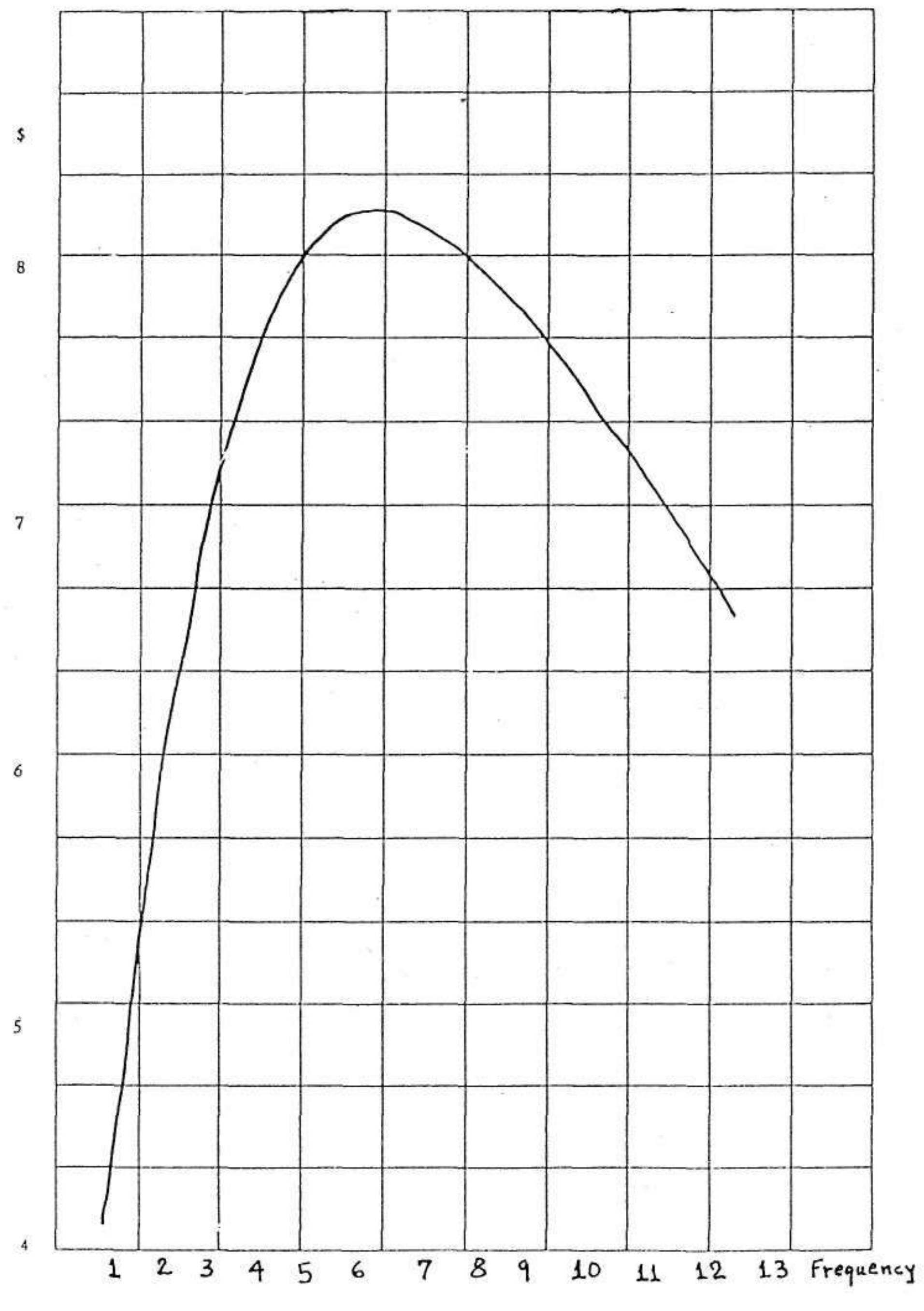


FIGURE 3

Net Gain as a Function of Budget, Given Frequency

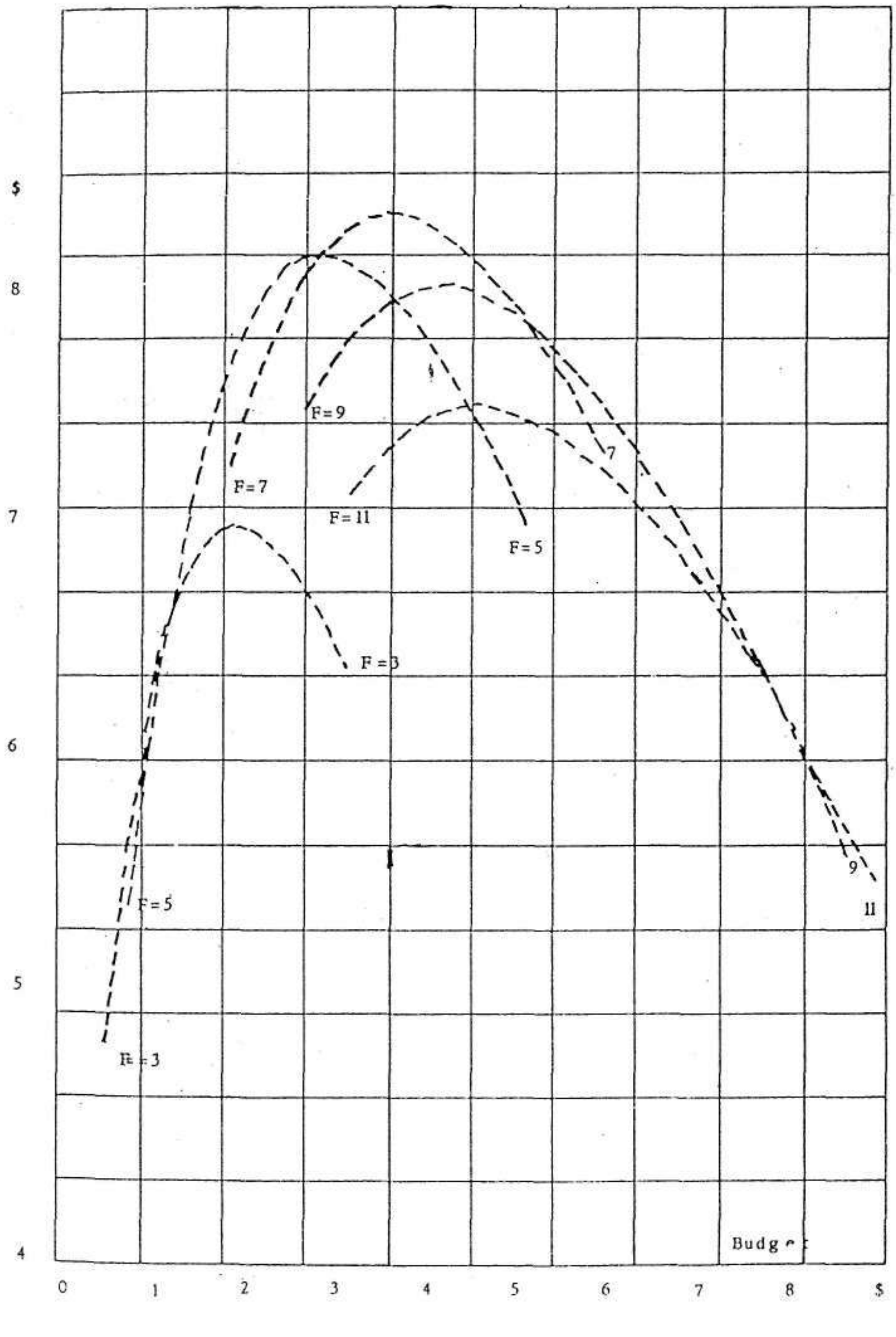
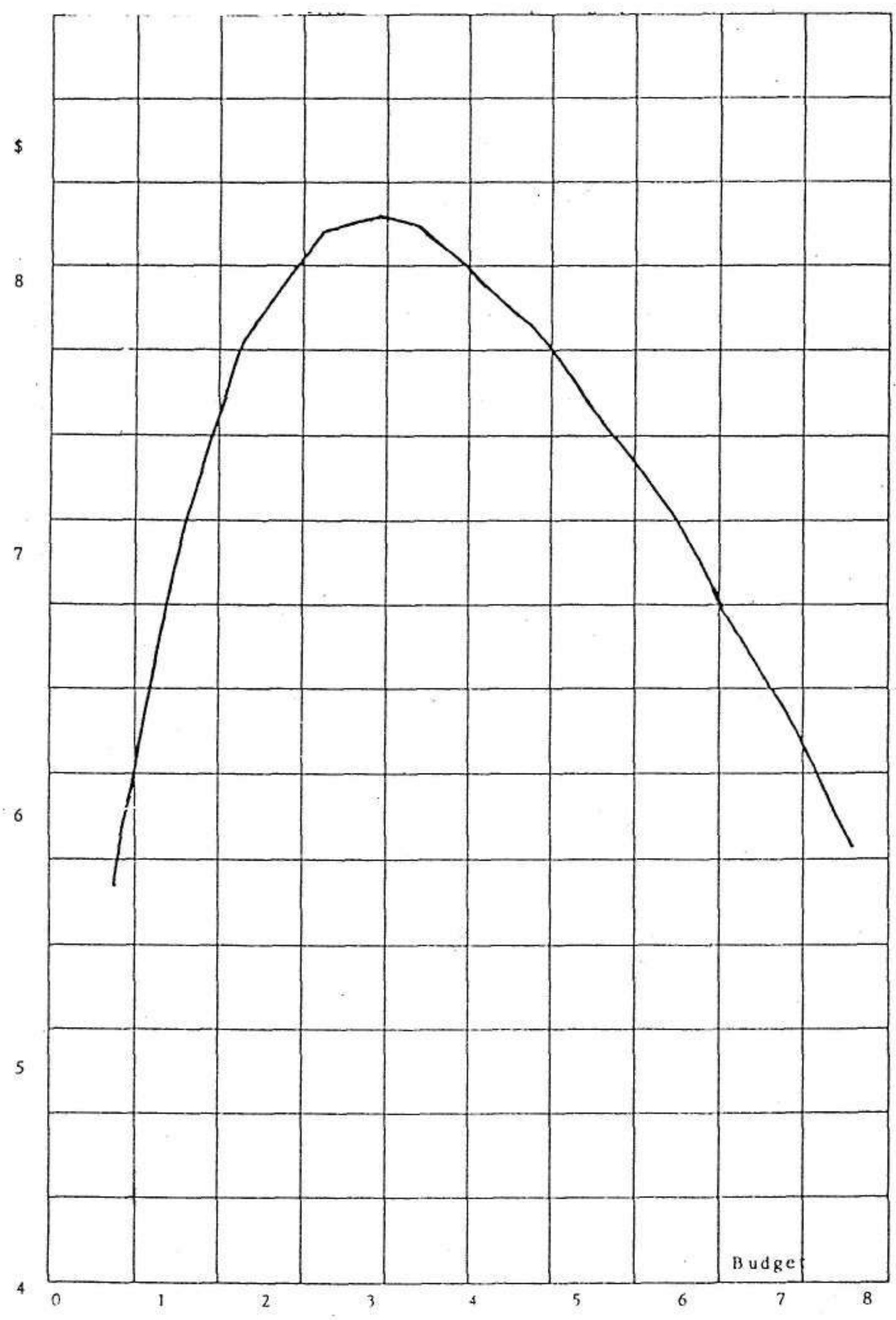


FIGURE 4

Net Gain as a Function of Budget, Given the Corresponding Optimal Frequency



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