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THE ROLE OF MULTI - DIMENSIONAL DECISION MODELS: A CRITICAL REVIEW AND AN APPLICATION

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Abstract

The paper reviews the applicability of approaches to multiple criteria decision making to aiding in the selection of a preferred option from a list of alternatives, and compares these approaches in detail from a theoretical standpoint. It is concluded that Analytic Hierarchy Process (AHP) is the approach best suited to this problem and the most widely used in practice. The paper goes on to present the Analytic Hierarchy process, together with its application to a multidimensional problem such as the systematic evaluation of alternative stock market choices.

1. Introduction

Decision makers are often puzzled when they have to choose among alternative solutions to a problem that:

— requires multiple criteria to be satisfied,
— has a number of alternative solutions available, and
— the ultimate outcome is uncertain even after the final solution is chosen.

The complexity of a problem leads to the evaluation of all alternatives against all criteria. This often results in different "optimal" solutions to emerge for different criteria. Identifying the criteria available and the alternatives is often problematic as it may not be possible to consider all criteria that are relative to the dicision and all alternatives available to the decision-maker. Finally, even after the identification of the criteria and alternatives and the evaluation of alternatives the problem is still puzzling as the final outcome of a specific choice is uncertain. Consequently, in order to systematically analyze such a problem, the following steps are required to be taken:

(1) Specification of all alternatives so as to be comparable to each other.

(2) Definition of all criteria in a way that allows for all alternatives to be compared to each other in respect to every criterion.

(3) Evaluation of criteria on the basis of goals persued or the purposes of the deicision maker.

The purpose of this article is to present the Analytic Hierarchy Process and its application in evaluating alternative stock market choices. The model allows the user to change the weights given to criteria and come up with an "optimal" solution for each set of weights. The model has been implemented on a computer for the use of the Operational Research Sector of the Commercial Bank of Greece.

2. The role of decision models in the evaluation of competing alternatives

A review of the relevant international literature, Huber (1974); Klee (1971); Roy & Vincke (1981); Zionts (1981); Schoemaker and Waid (1982); Dawes (1971); Fishburn (1971); Evans (1984); Buchanan and Daellenbach (1987); etc.) reveals that, until recently, two major approaches were suggested for handling problems of multicriteria decision making:

(1) Models whose solution is determined by the optimisation of a utility function (e.g. additive or multiplicative).

(2) Interactive models, which are either based on heuristics or on mathematical algorithms, depending on the information available to the decision maker. The explosive growth of microprocessing in recent years has contributed to the rapid development of this type of models.

By an additive model the weights and single attribute ratings are combined into an overall evaluation of each alternative as follows:

$$V_i = \sum W_j S_{ij}$$

Where V_i is the overall evaluation of alternative i;

W_j is the weight assigned to criterion j reflecting its "relative" importance to the other criterion;

S_{ij} is the score of the ith alternative with respect to the jth criterion.

Dawes (1971) and Keely and Doherty (1972) used this model to analyze the decision process of a university graduate Admissions Committee.

The multiplicative models often are presented as Conjunctive or Disjunctive models. For the conjunctive model the formulation involved the following equation:

$$U = \prod_{n=1}^{N} X_n^{a_n}$$ and the log-transformed model

$$\log U = \sum_{n=1} a_n \log(X_n)$$

Where: X_n are measures of the degree to which the alternative has been evaluated according to the nth criterion and α_n are weighting factors for the variables estimated using i.e. multiple regression.

For the disjunctive model the formulation involved the following equation:

$$U = \prod_{n=1} [1/(K_n - X_n)]^{b_n}$$ and the log-transformed model

$$\log U = \sum_{n=1} -b_n \log(K_n - X_n)$$

where K_n is some arbitrary value (see Einhorn, 1971); Goldberg, 1971).

These multiplicative models together with the linear additive model were used by Einhorn (1971) for two separate tasks. The first task involved a job preference decision problem. The second task consisted of choosing among applicants to graduate school in psychology; Comparing the two multiplicative models to the linear model he concluded that:

(1) The average accuracy of each of the two non-linear models relative to the linear did not change as a function the number of cues provided the judges and

(2) The use of the conjunctive model was especially prevalent, roughly equal to the linear model, while the use of the disjunctive model decreased considerably.

Goldberg (1972) designed a study to extend Einhorn's findings to a much larger and more representative set of data, for studying medical diagnosis. He compared the linear and non-linear models and his findings were in direct contradiction to those of Einhorn. He found that the linear model generally provided a better representation than did either the conjunctive or disjunctive models. To see why this happened he examined ten methological differences between Einhorn's and his study i.e.

(1) The type of task (job preferences vs differential diagnosis).

(2) The number of cases (30 vs 861) etc, and pointed out, "... in addition, future investigations who compare the conjunctive or disjunctive models with the linear models should:

a) Use reasonable large and representative samples of experimental protocols,

b) include enough "control" models so as to make the findings as unambiguous as possible and

c) pay special heed to the enormous problems involved in the use of models involving logarithmic or other nonlinear transformations of the original cues".

Tests of conjunctive or disjunctive models as well as linear additive models have been also reported by Huber (1974). He compares the models with respect to their ability to predict decisions and evaluations. The results indicate that the linear model did well. The conjunctive model proved as good as the linear additive, while the disjunctive model was the worst.

The procedure in an interactive method includes two alternative phases, the calculation and the decision-making phase i.e. conversation between the analyst and the decision maker.

In the calculation phase the analyst selects an action to put to the decision maker during the second phase. During the decision-making phase the decision maker examines the results of the calculation phase and by doing so he becomes able to give supplementary information about his objectives. The additional information is then introduced into the model in the next calculation phase. The best compromise is reached at after a certain number of cycles (each cycle includes the calculation and the decision making phases). The majority of interactive methods developed concern multiobjective linear programs. These programs require several, simultaneous objectives and have the advantage of accurately representing the real multicriteria nature of certain situations. The reader is referred to Buchanan and Daellenbach (1987), Evans (1984), Zionts (1981).

Saaty (1980) has proposed an alternative approach to this type of problem: the Analytic Hieararchy Process (AHP). The main characteristics of this method can be summarised as follows:

(1) Multidimensional problems are structured according to a hierarchy and can thus be handled more effectively;

(2) Priority theory is utilised at each level of the hieararchy through the performance of exhaustive pairwise comparisons;

(3) In contrast to other methods, the AHP allows the decision maker to identify and resolve easily inconsistencies in his evaluation of criteria and alternatives.

Through the hierarchical structure of the model, the general goal, which reflects the whole purpose of the analysis, is divided into subsidiary goals, so that its fulfillment is substituted by that of its components. The hierarchy is used both in structuring and in solving the complex system, in such a way as to achieve the following:

(1) Each decision element is dominated by all the elements of the immediately higher stratum and dominates all the elements in the immediately lower stratum;

(2) The decision elements at each stratum of the hierarchy are assessed independently from those at other strata.

The criteria and alternatives are considered separately through pairwise comparisons and appropriate priorities are assigned to them. The utilisation of pairwise comparisons is extremely significant, as in decision making problems it is usually impossible to select one altenative straightaway, since there is seldom one alternative which is preferable in terms of all the criteria. On the contrary, it is a common occurence that certain options are considered better than others in terms of some criteria, while others are considered prerefable in therms of the remaining criteria. This difficulty is resolved through the use of pairwised comparisons of alternatives in terms of single criteria.

Despite the criticism that multi-dimensional methods have received, some of them are widely used. The additive model is the earliest and probably the most widely used method. The multiplicative can be considered as a modification of the additive model, and has been proposed in order to overcome some of its weaknesses. The AHP, is a later development and has recently become increasingly popular. It has been applied to a wide range of problems. The areas in which AHP is applied are diverse and numerous (Economic/Management problems, Political problems, Social problems). An number of specific applications of AHP are listed below:

— Planning: Emshoff and Saaty (1982).
— Forecasting: Saaty and Gholamnezhad (1981), Cook, Falchi and Mariano (1984).
— Resource allocation: Arbel (1983), Lusk (1979), Sinuary-Stern (1984), Weiss (1990).
— Transportation: Saaty (1977).
— Marketing: Wind and Saaty (1980), Wind (1987), Dobias (1990).
— Finance: Vargas and Saaty (1981), Srinivasan and Bolster (1990).
— Education: Saaty and Rogers (1976).
— Banking: Arbel and Orgler (1990).
— Manpower selection and performance measurement: Lootsma (1980).
— Politics: Saaty and Bennett (1977).
— Public sector: Grizzle (1987).
— Sociology: Saaty and Wong (1983).

For a comprehensive review the reader is referred to Zahedi (1986) and Vargas (1990).

All these applications are decision problems and almost all involve rating decision alternatives for evaluation, selection, or prediction.

3. Advantages and disadvantages of the reviewed models

A variety of fairly good utility and interactive models are available, but few of them have been tested in large real world problems.

3.1. Utility models

Utility models have some deficiencies as decision aids such as:

(1) The decision maker is required to assess both the weights and utility functions according to his preferences on an interval scale. This may seem somewhat astonishing since the purpose of the model is to predict and prescribe decisions under certainty.

(2) The interval scale is subject to the preference of the decision maker. The utility models "force the decision maker to fit functions not truly representing his preferences", see Oppenheimer, K.R. (1978).

(3) When applying utility models, the question arises how to obtain comparisons of the utilities of decision alternatives, when each utility must take into account the contributions of many relavent factors. In order to solve this problem with additive utility models the assumption is made that the utility of a whole is equal to the sum of the utilities assigned to its parts. This assumption is not always representative of the real situation.

It is really difficult to answer the question "which of these models is the best". In every single case this depends on the particular decision situation. The utility models must be used with the greatest care because, some times the hypotheses on which they are based are unrealistic. If the hypotheses are realistic utility models work well.

3.2. Interactive over utility models

Interactive models have certain advantages over utility models.

(1) Interactive models do not require utility estimates on an interval scale as the decision makers's preference does not have to fit a particular functional form. They require local information about the utility function only in order to carry out the iterations. That is, the decision maker takes an active part every time an optimization problem has to be solved.

(2) Interactive models do not require explicit knowledge of the decision maker's utility function, but use it on an interactive basis with the decision maker by asking him certain questions "YES" or "NO".

(3) The process in interactive models can be considered as a learning process for the decision maker. As the procedure is carried out the decision maker may wish to return to an area that has previously been left out, and to include efficient solutions which have not been considered before.

(4) In addition, some assumptions such as utility independence are not required by the interactive models.

Interactive models compared to utility models show some deficiencies such as:

(1) Interactive models require a lot of the decision makers time as opposed to the utility models which require time only to assess the utility function.

(2) The working process of interactive models is more flexible, therefore it requires a more complicated program.

(3) The number of efficient solutions generated from these models is often too large and hence their effective analysis by the decision maker becomes cumbersome (Evans, 1984).

(4) When applying interactive models a large number of relatively difficult questions are posed and it is not always certain that the decision maker is in a position to answer them. The answer to the dilemma to use utility or interactive models depends on how much effort is required to develop an efficient interactive or utility model. Of cource utility models may be helpful in making simple routine decisions.

3.3. AHP over other models

AHP obtains to compute what is computed with the other methods of ratio scaling, with the advantage that this method faces the complex system with an efficient way structuring this system hierarchically. That is, this method is easier, simpler and more clear to identify the interrelated components of the problem and general to analyze the system. Although it is not generally presented as such by proponents of AHP in many applications the underlying model is an additive weighted similar to that presented above. The two approaches differ both in theory and in practice in the assessment of criteria weights ans scores. Saaty seems to resist this interpretation, but it is one which has frequently emerged by Belton (1986), Belton and Gear (1983) and Kamenetzky (1982).

AHP can be used to identify inconsistences. The other methods do not seem to tell us an essential way of estimating the consistency.

AHP, through the priority theory, permits decision makers to discuss the reasons for their estimates, to arrive at an agreement and to make a reconcilation if it is necessary.

A basic drawback in the use of the AHP is the effort required to complete all pairwise comparisons in large hierarchies. As the size of the hierarchy increases, the number of required pairwise comparisons increases exponentially. Harker (1987) developed a method (the Incomplete pairwise Comparison-IPC) in order to reduce this effort by ordering the questions in decreasing informational value and by stopping the process when added value of questions decreases below a certain level. Millet and Harker (1990) proposed an approach which can greatly enhance the effectiveness and attractiveness of the AHP for complex decision making problems with large hierarchies, through effective elicitation process. Other drawbacks of the method are presented in the conclusions of this paper.

The relationship between the results obtained from the AHP method with the additive value function is investigated by several researchers. Kamentzky (1982) compared the two approaches from a theoretical standpoind. Belton and Gear (1983) have commented on the specific problem of weight assessment. The paper by Belton (1986) goes on to compare these approaches in detail from both a theoretical and a practical standpoint. He found that the greatest weakness of the additive value function method is its failure to incorporate systematic checks on the consistency of judgements and the greatest weaknesses of the AHP are the ambiguous questioning procedure about criteria weights and the strong assumption of a ratio scale for the measurement of scores. All methods seem to suffer from drawbacks of one kind or another. Some times which is the best depends on the particular situation. A lot of methods are still included complicated mathematical models, which is resulted those methods to be inapplicable. This is particular the case in multiplicative utility models and part in interactive models. Assuming that the decision makers have the appropriate experience to weigh the significant criteria and to judge the alternatives, AHP method is comprehensive, systematic, insensitive to error in estimates provided by the decision maker, and the most easier to carry out the numerical calculations.

4. Brief description of the analytic hierarchy process

At this point, it is appropriate to present briefly the steps followed by the Analytic Hierarchy Process, so that a complete picture of the method can be formed.

During the first phase of the process, the problem is divided into different hierarchical strata, starting from the highest and moving down to the lowest, where the solution will eventually be determined. After the formulation of the hierarchy, priority theory is applied to the elements of each stratum separately, as follows:

Suppose that the relative importance (S_{ij}) of each pair of decision elements has been elicited from the decision maker. S_{ij} is the numerical value representing the relative importance of elements A_i and A_j. A scale of 1 to 9[1] has been suggested for the quantification of the relative importance of elements at each stratum of the hierarchy. Thus,

— if A_i and A_j are equally important, then $S_{ij} = 1$
— if A_i is more important than A_j, then $S_{ij} > 1$
— if A_i is less important than A_j, then $S_{ij} < 1$.

Table 1 contains the values S_{ij} representing the relative importance of decision elements determined through a series of pairwise comparisons, following the method described above.

Matrix A is defined as the $n \times n$ matrix of priorities S_{ij} contained in Table 1. Thus,

(1) if the elements of matrix A are consistent and defining W as the vector of relative weights $W_1, W_2, \ldots, W_n$ corresponding to the n decision elements, then the following is true

$$AW = nW \tag{1}$$

This means that matrix A has an eigenvalue equal to its dimension (since its rank) is 1) and the elements of the eigenvector W represent the relative priorities of the n decision elements.

(2) if matrix A contains inconsistencies, then (1) is modified to

$$AK = \lambda_{max} \tag{2}$$

where λ_{max} is the largest eigenvalue of A and the elements of the eigenvector K are estimates of the actual relative weights $W_1, W_2, \ldots, W_n$. Problem (2) is the generalised formulation of the eigenvalue approach to setting priorities.

The consistency of the elements of matrix A can be verified by checking whether relationship (3) holds.

$$S_{ik} = S_{ij} * S_{jk} \tag{3}$$

This relationship implies that it is sufficient to know one row of matrix A to construct the remaining entries. If the elements of matrix A are consistent, then

$$\lambda_{max} = n$$

where n is the dimension of the matrix. The closer λ_{max} is to n, the more valid are the estimates of the relative weights. For matrices where $S_{ij} = 1/S_{ji}$ holds, the largest eigenvalue is always greater than or equal to the dimension of the matrix. If the results are not consistent, the decision maker may review his assessment of the entries of matrix A, thereby improving consistency. The relationship

$$\alpha = \frac{\lambda_{max} - n}{n - 1}$$

is an appropriate index of consistency. If the value of α is less than or equal to 0.1, then the priorities elicited from the decision maker can be considered adequately consistent.

Table 1

Values of S_{ij}

Elements	A_1	A_2	A_3	......	A_n
A_1	1	S_{12}	S_{13}	.....	S_{1n}
A_2	S_{21}	1	S_{23}	.....	S_{2n}
A_3	S_{31}	S_{32}	1	.....	S_{3n}
A_n	S_{n1}	S_{n2}	S_{n3}	.....	1

5. Description of the application

This part of the article provides a detailed description of the methodology used and the results obtained by our application. Great emphasis is given to the scheduling and functioning of the model of the AHP, as our main purpose is twofold: to present the operation of the model in a simple application on the one hand and to underline the advantages of the computerization of the whole process. Computerization provides assistance to the decision maker as it simplifies computing procedure and determines the optimal choice, given the preferences of the decision maker.

The application refers to the accurate and systematic evaluation of alternative investments in securities of Greek commercial banks on the basis of simple, both quantitative and qualitative criteria. A simple hierarchical model, consisting of three levels, was constructed as following:

— the highest stratum is a single decision element, i.e. the overall assessment of the problem,
— the decision elements on the immediately lower stratum are the criteria according to which the selection will be performed,
— the third stratum consists of the set of alternative stock options, each of which is evaluated according to each criterion of the higher (second) stratum (see figure 1).

The ctiteria considered in the application of the model are:

(1) *Riskniness of stock:* The index of variability of the share price is used as an approximation of its riskiness. This index takes the form:

$$v = (\sigma/\mu)\%$$

where μ is the mean value and σ the standard deviation of the share price. Month-end values from January 1982 to December 1987 (72 observations) were used as our data set.

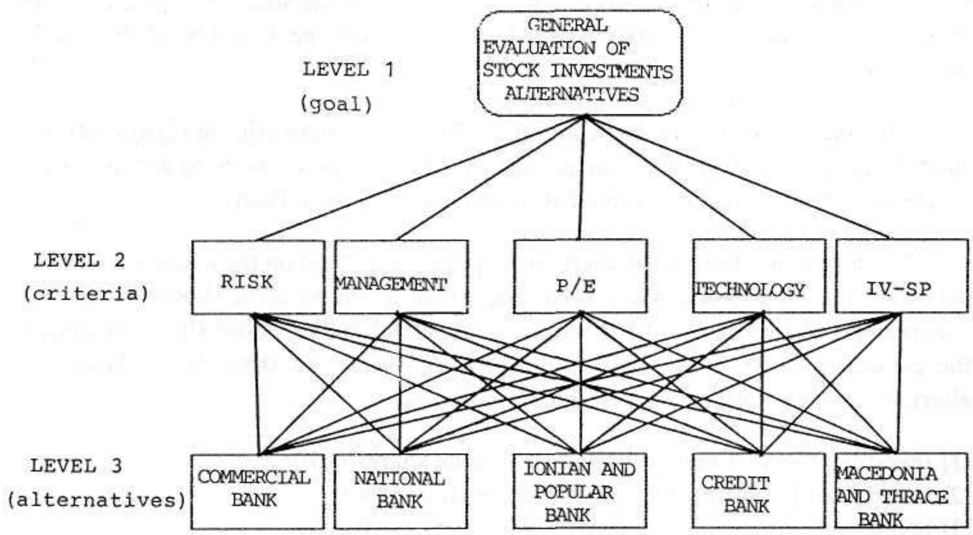

Fig. 1. Hierarchical three level model for the evaluation of stock investments alternatives.

(2) *The effectiveness of bank management:* This is evaluated on the basis of the ability of bank management to fulfill both the major and the secondary objectives of the bank, such as retaining the market share of the bank, expanding loans, etc.

(3) *The P/E relation:* This relation depicts the ratio

$$\frac{\text{Price per Share}}{\text{Earnings per Share}}$$

It has been evaluated using data from the Monthly Bulletin of the Athens Stock Exchange (December 1987).

(4) *The technological level of the bank's operations:* This is evaluated on the basis of the extent to which (i) branches belonging to the banks's network and (ii) types of transactions have been integrated into an online computer system.

(5) *The deviation of shares prices from its intrinsic value (IV-SP):* The intrinsic value of shares was estimated using a software package ISP (1988) written at the Operational Research Sector of the Commercial Bank. This package processes published data for each firm and determines the intrinsic value of a share, which is justified by the objectively measured state of the corresponding firm. The intrinsic value is used as an indicator of whether it is worthwhile buying or selling a particular share: if the intrinsic value is greater than the price of the share, then the decision maker would expect to gain by buying it - if it is lower, he would be likely to gain by selling it.

The assessment of each bank in terms of the qualitative criteria (2) and (4) was carried out on the basis of the conclusions reached by a special working group headed by the Director of the Operational Research Sector of the Bank.

The definition of the set of alternative options was based on the initial assessments determined by the Intrinsic Value model. Specifically, the Analytic Hierarchy Process was performed on the set of five Greek commercial banks whose shares displayed the greatest positive deviation between intrinsic value and share price. Thus, the alternative stock options were defined as follows:

(1) investment in Commercial Bank of Greece shares (CB)
(2) investment in National Bank of Greece shares (NB)
(3) investment in Ionian and Popular Bank shares (IO)
(4) investment in Credit Bank shares (CR)
(5) investment in Bank of Macedonia and Thrace shares (MT)

The relative importance of one decision element compared to another in terms of a quantitative criterion, such as those derived from published financial data (e.q. the P/E relation), has been quantified according to the ratio:

$$S_{ij} = \frac{AM(A_i)}{AM(A_j)}$$

where

— S_{ij} is the relative importance of A_i compared to A_j
— $AM(A_i)$ is the absolute value of decision element A_i
— $AM(A_j)$ is the absolute value of decision element A_j

Table 2 contains the relative importance of each criterion compared to all the others individually, as determined by a special working group.

Table 2

Pairwise comparisons of criteria relevant to the optimal selection of stocks

	Risk	Management	P/E	Technology	IV – SP
Risk	1	2	1/3	3	3
Management	1/2	1	1/3	3	2
P/E	3	3	1	5	5
Technology	1/3	1/3	1/5	1	1/2
IV – SP	1/3	1/2	1/5	2	1

The vector K of relative weights estimated for each criterion on the basis of the entries of Table 2 is:

Risk	22.6%
Management	15.6%
P/E	46.4%
Technology	6.4%
IV – SP	9.0%

The consistency index is 96.75% and the corresponding largest eigenvalue λ_{max} is equal to 5.13 which is very close to the consistency value 5. It is obvious from the results that the value of the P/E ratio is the most important criterion, while the technological level is the least significant. It should be noted that if the decision maker were not satisfied by these results, he could easily review his priorities and adjust them accordingly. This can be accomplished easily using the software package developed for this purpose.

Table 3

Comparison of alternative stock options in terms of the criteria

Risk[1].

	CB	NB	IO	CR	MT
CB	1	0.47	0.74	1.28	0.92
NB		1	1.58	2.72	1.95
IO			1	1.72	1.23
CR				1	0.72
MT					1

Management

	CB	NB	IO	CR	MT
CB	1	3	2	1	4
NB		1	1/2	1/3	2
IO			1	1/2	2
CR				1	4
MT					1

P/E[2].

	CB	NB	IO	CR	MT
CB	1	0.77	2.17	1.44	1.86
NB		1	2.83	1.89	2.43
IO			1	0.67	0.86
CR				1	1.29
MT					1

Technology

	CB	NB	IO	CR	MT
CB	1	1	1/3	1/3	3
NB		1	1/3	1/3	3
IO			1	1	5
CR				1	5
MT					1

Iv – SP

	CB	NB	IO	CR	MT
CB	1	5	2	5	3
NB		1	1/4	1	1/2
IO			1	3	2
CR				1	1/3
MT					1

Table 3 contains the relative ratings of alternative stocks in terms of each criterion. For each matrix, the largest eigenvalue and corresponding eigenvector were calculated and are presented in Table 4. Each element in an eigenvector represents the relative rating of an alternative in terms of the corresponding criterion.

Table 4
Priorities of alternatives in terms of each criterion

CRITERION ALTERNATIVE	Risk	Management	P/E	Technology	IV – SP
CB	15.8	32.1	25.0	12.9	43.1
NB	33.5	11.1	32.6	13.0	7.6
IO	21.2	17.2	11.5	34.3	26.0
CR	12.3	32.2	17.4	34.3	7.4
MT	17.2	7.4	13.5	5.5	15.9
λ_{max}	5	5.04	5	5.06	5.06
consistency index	100%	99%	100%	98.5%	98.5%

The priority of each alternative on each criterion is weighted by the relative importance of the corresponding criterion and the sum of the weighted priorities indicates the overall rating of each alternative taking into account all available information, i.e. $W = D*K$, where

— W is the vector of overall priorities of the alternatives
— D is the matrix of the priority of each alternative in terms of each criterion
— K is the vector of priorities of the criteria.

Thus, the "priorities" of the alternatives turn out to be as follows:

Alternative	**Priority**
Commercial Bank	24.9
National Bank	26.0
Ionian and Popular Bank	17.3
Credit Bank	18.7
Bank of Macedonia and Thrace	13.1

These results indicate that, according to the data used, the optimal choice of purchase is in National Bank shares, followed by shares in the Commercial Bank. The data used are acceptable in terms of consistency.

It should be noted that the application presented does not necessarily provide objective results as, on the one hand, it fails to incorporate all factors affecting share prices and, on the other, is affecterd by the subjective judgment of the decision

—maker (in this case the special team) in attaching "importance coefficients" to the criteria. This implies that, in order to provide better results, the model requires considering a great number of criteria - factors affecting share prices (directly or indirectly, economic, political, etc.) and more than three hierarchical levels (Saaty et al, 1980). Using three levels was chosen in order to demonstrate the usefulness of the method in decision - making and the advantages achieved once the procedure is fully computerized.

6. Conclusions

This paper has presented the AHP in relation to the multicriteria evaluation of alternative stock market choises. This model has been implemented on a computer and the corresponding software package can be run on any IBM or IBM compatible machine.

The AHP was presented in the context of a relatively simple decision problem, in order to facilitate the comprehension of the technique. The ultimate objective of this presentation is to explore the potential of the method in more complex applications to decision problems.

The AHP model belongs to a series of recent developments in the area of multicriteria decision making and has already been successfully applied to a number of different contexts (Saaty, 1980; Sahedi, 1986; Vargas, 1990 etc). In most of these cases, the criteria used were qualitative. The AHP is a technique for the subjective judgement of alternative options and, as a consequence, it relies heavily on the experience of the decision maker and his ability to perform qualitative comparisons. A brief overall assessment of the method reveals the following:

Firstly, the successful application of the technique requires that the decision maker has the neccessary experience to weigh the importance of the criteria and to compare each pair of alternatives in terms of each criterion. Another drawback of the technique (which is also true of other similar techniques) is the fact that it is impossible to check its results objectively. For example, it is impossible to check statistically the importance of criteria and the valibity of performed comparisons, or the extent to which the available data are representative of reality (although certain statistical measures, such as the Root of Mean Square Deviations, have been proposed for this purpose).

However, the abovementioned drawbacks are offset by the advantages of the method. The latter include the following:

(1) The decision problem is dealt with effectively through the use of hierarchical strata.

(2) The decision maker is allowed to recognise and deal with inconsistencies in his judgements. Any inconsistency identified in his evaluations can be resolved by returning to the previous phase of the process and performing the comparisons anew.

(3) The technique is simple to use and allows the decision maker to review his judgements, should the results not satisfy him.

(4) Through the utilisation of pairwise comparisons, the AHP facilitates the comparison of dissimilar alternatives, which under different circumstances would be impossible to compare.

(5) Finally, the technique makes possible the comparison of alternatives on qualitative criteria, through the use of appropriate quantification scales.

Summarising, the above reinforce the view that the Analytic Hierarchy Process is a significant tool for dealing with multidimensional problems of choice among many alternatives on the basis of multiple criteria.

NOTES

1. A variety of scales is suggested in existing bibliography. However, Saaty (1980) has proposed the 1 to 9 scale as the most appropriate for the specific problem.

2. In comparing banks according to "Risk", it was assumed that shares whose prices displeyed low variability were preferable to shares with highly variable prices. This is considered rational from a "conservative" viewpoint.

3. In comparing banks according to "P/E", it was assumed that higher values of the ratio were preferable to lower values. This was thought to be rational in the bank context, where the shares are acquired in view of their prospects for capital gains.

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