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MAXIMIZING THE RELIABILITY OF MARKETING MEASURES UNDER BUDGET CONSTRAINTS

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Abstract

Generalizability theory provides a framework for examining the dependability of measurements in marketing research. When limited resources are available determining the optimal number of observations to use in a marketing measurement design that will maximize reliability is not a simple task. This paper presents a method for determining the optimal number of observations to use in fully-crossed, univariate and multivariate two- and three- facet measurement designs when resource constraints are imposed. (JEL C61, M31).

1. Introduction

In recent years the theory of generalizability has gained increasing attention with marketing researchers, as evidenced by the growing number of studies in the literature which apply it (Peter, 1977; Peter, 1979; Berhman and Perreault, 1982; Rentz, 1987). Generalizability (G) theory (Cronbach, Gleser, Nanda, and Rajaratnam, 1972) is a theory that provides a framework for examining the dependability of measurements in marketing research. G theory extends classical reliability theory most notably by recognizing and estimating the magnitude of multiple sources of errors in measurements. Rentz (1987) reviewed the major concepts in G theory and illustrated its use as a comprehensive method for designing, assessing, and improving the dependability of marketing measures. Clearly, the greatest contribution of generalizability theory to marketing research lies in its ability to model a remarkably wide array of measurement conditions. Unfortunately, when limited resources are available the ability to design measurement studies tht maximize reliability is not a simple task. The purpose of this paper is to present a method that can be used to determine the optimal measurement design when cost constraints are imposed on the design.

2. Review of Basic Generalizability Theory Concepts and Measurement Issues

In generalizability theory there is an important distinction between generalizability (G) studies and decision (D) studies. G studies are associated with the development of a measurement procedure, whereas D studies apply the procedure in practical terms (Shavelson and Webb, 1981). In fact, according to Rentz (1987), the greatest benefits of generalizability analysis are derived when modifications to a measurement procedure are analyzed and an acceptable design is chosen relative to maximizing the reliability within cost or other practical constraints. Thus, if the results of a G study show that some sources of error in the design are very small, then a decision maker may reduce the number of levels of that facet (e.g., occasion of observation), or may even ignore that facet in a D study. This permits a smaller and less costly design for the D study than that used in the G study.

A major contribution of generalizability theory, therefore, is that it permits a decision maker to pinpoint the sources of measurement error in the design and increase the appropriate number of observations accordingly in order to obtain a maximum level of reliability (Shavelson, Webb, and Rowley, 1989).

For example, in a study of the dependability of measures of brand loyalty (Peter, 1979), the investigator considered items and occasions to be important factors that could lead to the undependability of the measurement procedure. Thus, the variance components for a person by items by occasions ($\rho \chi i \chi o$) fully crossed design were estimated using a 10 item brand loyalty scale administered to 100 persons on 3 occasions. One question that a researcher can determine is whether the reliability of the measurement procedure can be increased by adding more occasions or items.

The estimated variance component for each source of variation in the above example G study of brand loyalty scores are presented in Table 1. From these estimated variance components, a generalizability coefficient, analogous to the classical reliability coefficient, can be calculated by dividing the estimated person variance component by the estimated observed-score variance. As can be seen in Table 1, occasions of measurement are a substantial source of error variation. The item variance is relatively small indicating that the items used to measure brand loyalty are providing consistent information. This is also reflected in the small variance components of the person by item interaction and the item by occasion interaction. Clearly, the number of occasions of measurement has the greatest effect on generalizability, whereas the number of items has little effect. For example, using 10 items and 1 occasion will produce an estimated relative generalizability coefficient of p^2 = 0.84, compared to p^2 = 0.79 when using 5 items and 1 occasion. However, when using only 5 items and 3 occasions produces an estimated generalizability level of p^2= 0.91 [For a formal development of the generalizability (G) coefficent and variance component estimates see Shavelson and Webb (1981), Rentz (1987), and Marcoulides, (1989a)].

The ability to design subsequent D studies more effeciently on the basis of information from the G study is clearly one of the major advantages of generalizability theory for marketing researchers. By trading off desired levels of reliability and costs researchers can design optimal D studies. Unfortunately, while it is important to have large coefficients of generalizability, such are not always possible when conditions of scarce resources are present. The question then becomes how to maximize the generalizability coefficient within a prespecified amount of limited resources. For example, in a one-facet person by item ($\rho \chi$ i) design, the question of satisfying resource constraints is simple. Choose the most items that will give maximum generalizability without going over the available budget. Unfortunately, when other facets are added to the design, obtaining a solution can become quite complicated, especially since each decision will produce a different costing D study design.

Marketing measurements also often involve multiple scores in order to describe individuals' preferences (Peter, 1979). For example, an instrument designed to measure brand loyalty might use subtests to measure two different dimensions of loyalty, or an instrument designed to measure consumer problems relating to food products might use subtests to measure five different dimensions: physiological, sensory, activities, buying and usage, and psychological/ social (Tauber, 1975). Under such measurement conditions, in order to examine the multiple dependent scores simultaneously, a multivariate generalizability analysis is essential and the univariate procedures described by Rentz (1987) must be extended to include multivariate designs.

In multivariate generalizability theory the notion of multifaceted error variance is basically extended to include all types of multivariate designs. As much, multiple scores are treated simultaneously and the matrices of variance and covariance components provide the essential information for deciding whether the multiple scores in the measurement battery should be treated as a profile or a composite as opposed to separate scores. Additionally, using the matrices of variance and covariance components, and using the multivariate extension of the univariate generalizability coefficient developed by Joe and Woodward (1976), a decision maker can obtain the dimensions of scores which provide maximum generalizability. Unfortunately, while Joe and Woodward's (1976) procedure for calculating generalizability coefficients in multivariate designs produces the coefficient with maximum generalizability, this procedure does not take into account any budgetary constraints that might be imposed on the design (i.e. the generalizability coefficient is obtained for an unconstrained solution).

It appears, therefore, that determining the optimal design to use in a univariate or a multivariate multifaceted measurement design is not a simple task once the number of facets is more than one. The purpose of this paper is to present a general procedure for determining the optimal number of observations to use that will maximize coefficients of generalizability in both univariate and multivariate multifaceted designs. The paper can be considered an extension on the work of Cronbach et al. (1972), Joe and Woodward (1976), Rentz (1987), and Shavelson and Webb (1981). Basically, for univariate and multivariate multifaceted designs, a simple procedure is presented to determine the optimal number of observations to use when cost constraints are imposed on the design. We hope, by providing a clear and understandable picture of the procedure, that the practical applications of this method will be adopted for optimizing measurement designs in marketing research.

3. Selecting the Optimal Number of Observations in Measurement Designs

3.1. The Two-Facet Univariate Case

Consider the two-facet person by item by occasion ($p \times i \times o$) study of the dependability of measures of brand loyalty presented by Peter (1979). In order to find, for example, the maximum relative generalizability, without violating budget constraints, a certain number of observations for each facet must be selected.

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The problem is a nonlinear optimization problem in which the decision variables are the number of items (n_i) and occasions (n_o). The objective of the optimization is to maximize the generalizability coefficient (ρ^2_δ) defined as:

$$\rho^2_\delta = \frac{\sigma^2_p}{\sigma^2_p + \sigma^2_\delta} \tag{1}$$

where
$$\sigma^2_\delta = \frac{\sigma^2_{pi}}{n_i} + \frac{\sigma^2_{po}}{n_o} + \frac{\sigma^2_{pio,\, e}}{n_i n_o} \tag{2}$$

under the budgetary constraint that

$$c n_i n_o \leq \bar{c} \tag{3}$$

where

c = cost per item per occasion, and
$\bar{c}$ = total budget available.

Our task then is to find a set of values for n_i and n_o within the specified budget ($\bar{c}$) which maximize ρ^2. Since with respect to the optimization problem σ^2_p is a constant, the objective is to minimize σ^2_δ under the imposed budget constraint of (3).

To obtain a solution let us differentiate the Lagrange function

$$\min F(n_i, n_o, \lambda) = \frac{\sigma^2_{pi}}{n_i} + \frac{\sigma^2_{po}}{n_o} + \frac{\sigma^2_{p\ \ io,\, e}}{n_i n_o} - \lambda\,(c n_i n_o - \bar{c}) \tag{4}$$

with respect to n_i, n_o, λ and equate the derivatives to zero.

This yields:

$$n^*_i = \sqrt{\frac{\sigma^2_{pi}}{\sigma^2_{po}} \cdot \left(\frac{\bar{c}}{c}\right)} \tag{5}$$

$$n^*_o = \sqrt{\frac{\sigma^2_{po}}{\sigma^2_{p\ i}} \cdot \left(\frac{\bar{c}}{c}\right)} \tag{6}$$

Thus, if the total available budget for the study is \$50 and the cost per item per occasion is \$5.00, then the required number of items and occasions, using the variance components in Table 1 are $n^*_i = 2$ and $n^*_o = 5$ (since the values of n_i and n_o must be integers the solutions are always rounded to the nearest integer such that the budget constraint is satisfied). The maximum generalizability coefficient can then be obtained using equation (1), and in this example is equal to $\rho^2 = 0.90$. Of course, additional constraints, for example on the upper bounds of the number of occasions or the lower bounds of the number of items could easily be imposed on the problem before optimizing the solution. Such additional constraints might become important when, besides the budgetary constraints, a decision-maker is aware of other design restrictions. In this paper, however, we are only interested in budgetary constraints.

3.2. The Three-Facet Univariate Case

Consider a three-facet hypothetical measurement design. Salespersons' performance is measured on a 31-item job performance scale (like the one developed by Behrman and Perreault, 1982) independently by three supervisors on two occasions. The basic design in this hypothetical study is salespersons crossed with items (i), occasions (j), and supervisors (k) with n_i, n_j, and n_k number of observations for each facet.

Thus, the error function to be minimized is

$$\sigma^2_\delta = \frac{\sigma^2_{pi}}{n_i} + \frac{\sigma^2_{pj}}{n_j} + \frac{\sigma^2_{pk}}{n_k} + \frac{\sigma^2_{pij}}{n_i n_j} + \frac{\sigma^2_{pik}}{n_i n_k} + \frac{\sigma^2_{pjk}}{n_j n_k} + \frac{\sigma^2_{res}}{n_i n_j n_k} \tag{7}$$

in order to maximize ρ^2 from (1) subject to the constraint that

$$c n_i n_j n_k \leq \bar{c} \tag{8}$$

where

c = cost per ijk combination, and
$\bar{c}$ = total budget available for the measurement.

Once again, for our solution let us differentiate the Lagrange function $L(n_i, n_j, n_k, \lambda) = \sigma^2_\delta - \lambda(c n_i n_j n_k - \bar{c})$ with respect to n_i, n_j, n_k, and λ. By setting the results equal to zero we obtain at the optimal point:

$$\frac{\sigma^2_{pi}}{n_i} - \left(\frac{c}{\bar{c}}\right) \sigma^2_{pik}\, n_i = \frac{\sigma^2_{pj}}{n_j} - \left(\frac{c}{\bar{c}}\right) \sigma^2_{pjk}\, n_j = \frac{\sigma^2_{pk}}{n_k} - \left(\frac{c}{\bar{c}}\right) \sigma^2_{pij}\, n_k \quad (9)$$

or

$$\alpha(n_i) = \beta(n_j) = \Gamma(n_k)$$

and

$$n_i\, n_j\, n_k = \left(\frac{\bar{c}}{c}\right) \quad (10)$$

While no closed form solution can be found for the number of observations of i, j, and k, from (9) we observe that at the optimal point of the solution there is a special symmetric monotonically decreasing structure to the functions presented that can be used in a simple search procedure that converges quickly to the environment of the optimum solution for n_i, n_j, and n_k.

The proposed search procedure is as follows:

6(i) Start with any value of n_i, say $n_i^{(1)}$ and solve for $n_j^{(1)}$ and $n_k^{(2)}$ by the known equality (9), namely $\alpha(n_i) = \beta(n_j) = \Gamma(n_k)$.

(ii) Substitute the values of $n_i^{(1)}$, $n_j^{(1)}$, $n_k^{(1)}$ and calculate $n_i n_j n_k$.

(iii) If $n_i\, n_j\, n_k = \frac{\bar{c}}{c}$, the optimal solution has been obtained. Otherwise, one of two conditions holds; **Condition 1:** If $n_i\, n_j\, n_k < \frac{\bar{c}}{c}$, at least one variable must be increased to satisfy the equality $n_i\, n_j\, n_k = \frac{\bar{c}}{c}$ and, consequently all the variables must be increased to satisfy the necessary condition of optimality in (9). An upper bound on say n_i can be obtained by $n_i^{(1)} < n_i < \left(\frac{\bar{c}}{c}\right)\left(\frac{1}{n_j^{(1)}\, n_k^{(1)}}\right)$. **Conditions 2:** If $n_i\, n_j\, n_k > \frac{\bar{c}}{c}$, at least on variable must be decreased to satisfy the equality $n_i n_j n_k = \frac{\bar{c}}{c}$ and, consequently all the variables must be decreased to satisfy the necessary condition of optimality in (9). A lower bound on n_i can be obtained by $\left(\frac{\bar{c}}{c}\right)\left(\frac{1}{n_j^{(1)}\, n_k^{(1)}}\right) < n_i < n_i^{(1)}$.

(iv) Determine $n_i = \frac{\text{lower bound} + \text{upper bound}}{2}$, and use this value to obtain n_j and n_k as in step (i) - (at iteration r the values will be $n_i^{(r)}$, $n_j^{(r)}$, and $n_k^{(r)}$).

4. Illustrative Example Using Search Procedure

The following example will illustrate how the search procedure works to solve the three-facet case. Let us make use of the following estimated variance components: $\sigma^2_{pi} = 2$, $\sigma^2_{pj} = 3$, $\sigma^2_{pk} = 5$, $\sigma^2_{pij} = 1$, $\sigma^2_{pik} = 6$, $\sigma^2_{pjk} = 4$, and $\sigma^2_{res} = 2$. If the total budget available is $\bar{c} = 500$, and cost per ijk combination is $c = 10$, using the proposed procedure we can calculate the optimal number of conditions to use to obtain the largest coefficient of generalizability.

By arbitrarily setting $\alpha(n_i) = 0$ we get $n_i^{(1)} = 4.08$, and by substitution in a similar manner in (9) we get from $\beta(n_j)$, $n_j^{(1)} = 6.12$, from $\Gamma(n_k)$, $n_k^{(1)} = 15.8$, and $\frac{\bar{c}}{c} = 50$. This results in $n_i n_j n_k = 394.8 > 50$. Thus, $n_i^{(1)}$, $n_j^{(1)}$, and $n_k^{(1)}$ must be simultaneously reduced according to Condition 2.

Let us calculate the lower bound on $n_i^{(1)}$ by $\left(\frac{\bar{c}}{c}\right)\left(\frac{1}{n_j^{(1)}\, n_k^{(1)}}\right) < n_i < n_i^{(1)}$ which gives $0.52 < n_i < 4.08$. By the simple bisection on the bounds the new value of $n_i^{(2)}$ is $\frac{4.08 + 0.52}{2} = 2.30$. Using this value from $\beta(n_j)$ and $\Gamma(n_k)$ we get $n_j^{(2)} = 3.44$ and $n_k^{(2)} = 6.84$. Thus, $n_i n_j n_k = 54.07 > 50$ and once again, $n_i^{(2)}$, $n_j^{(2)}$, and $n_k^{(2)}$ must be reduced.

Calculating a new lower bound for $n_i^{(2)}$ we get $n_i^{(3)} = 2.21$, $n_j^{(3)} = 3.31$, and $n_k^{(3)} = 6.49$. This gives $n_i n_j n_k = 47.48 < 50$, and we see that if these values are rounded to the nearest integers we get a solution close to the optimal with $n_i = 2$, $n_j = 3$, and $n_k = 6$. This combination of the number of facets for ijk yields the largest coefficient of generalizability undee the imposed budget constraint (in this example $\rho^2 = 0.91$).

5. Selecting the Optimal Number of Observations. In Multivariate Designs

As previously indicated, marketing measurements also sometimes involve multiple dependent scores. Under such measurement conditions, a multivariate analysis must be selected to find the maximum multivariate generalizability coefficient. While there are some similarities between the optimization of univariate and multivariate designs, there are also some important differences that require we present the multivariate method of optimization in its entirety for the two- and three- facet example designs.

5.1. The Two - Facet Multivariate Case.

Consider the following multivariate two - facet crossed person by occasion by rater (p x o x r) design presented in Marcoulides (1989b). A salesperson's job performance measure consists of three major forms. Each form consists of items (e.g. for Form A these might include meeting deadlines, volume of work, accepting direction, etc.) and the score for each form is the sum across items on that form. In the generalizability study, two supervisors (raters) rate 35 salespersons on two different occasions on job performance.

Table 2 presents the results of the variance - covariance components obtained from a generalizability analysis on the job performance test data. In order to find the maximum generalizability coefficient without violating budget constraints, a certain number of observations for each facet in the multivariate design must be selected. It is important to note that changing the number of levels of a facet in a multivariate decision study has an effect on the estimate of error in the same manner as in the univariate case. The problem of finding what number of observations to use for each facet, therefore, just like in the univariate case, is a nonlinear optimization problem in which the decision variables are the number of occasions (n_o) and raters (n_r). The objective of the optimization, of course, is to maximize the multivariate generalizability coefficient (ρ^2), defined by Joe and Woodward (1976) as:

$$\rho^2 = \frac{\underset{\sim}{a}' V_p \underset{\sim}{a}}{\underset{\sim}{a}' V_p \underset{\sim}{a} + \underset{\sim}{a}' V_\delta \underset{\sim}{a}} \tag{11}$$

where

$$V_\delta = \frac{V_{po}}{n_o} + \frac{V_{pr}}{n_r} + \frac{V_{por,\,e}}{n_o n_r} \tag{12}$$

under the budgetary constraint that

$$c n_o n_r \leq \bar{c} \tag{13}$$

where

c = cost per rater per occasion,

$\bar{c}$ = total budget available,

V = a matrix of variance and covariance components estimated from the mean product matrices for each effect,

$\underset{\sim}{a}$ = the vector of canonical coefficients that maximizes the ratio of person variation to person plus error variation, and

V_δ = the relative error matrix (the multivariate analogue to the univariate error σ^2_δ).

According to Joe and Woodward (1976), the ρ^2 and $\underset{\sim}{a}$ for any design can be obtained by solving the following set of equations:

$$[\, V_p - \rho_\lambda^2 \, (V_p + V_\delta) \,] \, \underset{\sim}{a} = 0 \tag{14}$$

where the subscript λ refers to the characteristic root of (14) and $\underset{\sim}{a}$ its associated eigen vectors.

Thus, for each multivariate generalizability coefficient corresponding to a characteristic root in (14), a set of coefficients defines a composite of scores in the design. The number of composites defined by the coefficient is equal to the number of different measures entered in the analysis (in this example the 3 forms). By definition, the first composite will be the most reliable (Webb, et al., 1983).

Our task then is to find a set of values for n_o and n_r which maximize ρ^2 without violating the budgetary constraint (13) imposed. Note that although for a given number of observations of facets the maximum generalizability coefficient is obtained by finding the largest characteristic root of (14), in our constrained problem the number of observations is unknown (since that is what the optimization will actually determine). So the characteristic equations of (14) becomes an implicit function of λ (the eigenvalue) and the number of observations (like n_o and n_r). Therefore, because obtaining an exact solution of an implicit function is intractable, we propose a constrained search procedure that will result in a sufficiently good solution that is very close to the optimal. This procedure is based on the observation that an optimal solution, which provides the largest value possible for the G coefficent under the budget constraints imposed, can be obtained once all the elements in the variance- covariance matrix V_δ are minimized. Since with respect to the optimization problem V_p is a known constant matrix, our multidimensional objective is to minimize all components in the matrix V_δ under the imposed constraint that

$$c n_o n_r \leq \bar{c} \tag{15}$$

In the univariate case this minimization would be equivalent to minimizing σ_δ^2 (as in equation (2)) with the optimal values of n_o and n_r obtained (similarly to (5) and (6).

For the multivariate design, however, we have the matrix V_δ, with each element in the diagonal being equivalent to the univariate case presented in (2), while the off-diagonal elements include the covariances of the components. If we denote the elements in the V_δ matrix as δ_{jk}, our objective is to minimize all of them simultaneously. Thus, the minimization is equal to:

$$\text{Min } \delta_{11} = \frac{v_{po_{11}}}{n_o} + \frac{v_{pr_{11}}}{v_r} + \frac{v_{por,e_{11}}}{n_o n_r}$$

$$\text{Min } \delta_{12} = \frac{v_{po_{12}}}{n_o} + \frac{v_{pr_{12}}}{v_r} + \frac{v_{por,e_{12}}}{n_o n_r} \quad (16)$$

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

$$\text{Min } \delta_{mn} = \frac{v_{po_{mn}}}{n_o} + \frac{v_{pr_{mn}}}{v_r} + \frac{v_{por,e_{mn}}}{n_o n_r}$$

subject to the restriction that

$$c n_o n_r \leq \bar{c}\,, \quad (17)$$

where lower– case v is the variance or covariance component in the design.

Unfortunately, the optima for each δ_{jk} may not necessarily be the (global) optima. In fact, a global optima might not even exist. In other words, the values of n_o and n_r that might minimize each of these equations individually might not be the optimal for all of them together. Thus, a conflict might occur which can only be resolved by finding that combination of n_o and n_r that actually minimizes the entire set of equations in the best possible way. For our solution let us first assume that we can optimize V_δ with respect to each element δ_{jk} in the matrix independently [as can easily be observed, at any optimal point the constraint is satisfied as an equality, therefore, for the optimization $\frac{v_{por,\ e_{jk}}}{n_o n_r}$ is constant and can be ignored]. Denote that solution value by δ^*_{jk}, which basically becomes the goal of each δ_{jk} minimization. However, due to the possibility of a conflict occuring between that combination of n_o and n_r that actually minimizes the entire set of equations, we will consider only the deviations from the goal. The deviations from the goal are denoted as:

$$d_{jk} = \delta_{jk} - \delta^*_{jk} \geq 0 \qquad \text{for all j and k} \quad (18)$$

We then try to minimize the sum of these deviations in order to obtain values as close as possible to the intended goals, namely:

$$\text{Min} \sum_k \sum_j d_{jk} \quad (19)$$

subject to

$$\frac{v_{po\ jk}}{n_o} + \frac{v_{pr\ jk}}{n_r} - d_{jk} = \delta^*_{jk} \tag{20}$$

j= 1, .., m
k= 1, .., n

$$n_o n_r = N \tag{21}$$

(Note that for convenience we write $\frac{\bar{c}}{c} = N$)

and

$$n_o,\ n_r \geq 0 \tag{22}$$

For simplicity, let us eliminate the variable n_r by the equality constraint $n_o n_r = N$, (which gives $n_r = \frac{N}{n_o}$) and obtain the minimization of the sum of the deviations as in (19) subject to

$$\frac{v_{po\ jk}}{n_o} + \frac{v_{pr\ jk}}{N} \cdot\ n_o - d_{jk} = \delta^*_{jk} \tag{23}$$

along with (21) and (22).

Using the Lagrange relaxation we get:

$$L(\lambda, n, d) = \sum_k \sum_j d_{jk} - \sum_k \sum_j \lambda_{jk} \left(\frac{v_{po\ jk}}{n_o} + \frac{v_{pr\ jk}}{N} \cdot n_o - d_{jk} - \delta^*_{jk} \right) \tag{24}$$

where $\lambda = \{\lambda_{jk}\}$ for $j = 1, .., m;\ k = 1, .., n$
$d = \{d_{jk}\}$ for $j = 1, .., m;\ k = 1, .., n$.

By differentiation with respect to λ_{jk}, d_{jk}, n_o and setting the results equal to zero we get:

$$n^*_o = \sqrt{\frac{\sum_k \sum_j v_{po_{jk}}}{\sum_k \sum_j v_{po_{jk}}} \cdot N} \tag{25}$$

and

$$n^*_r = \sqrt{\frac{\sum_k \sum_j v_{pr_{jk}}}{\sum_k \sum_j v_{pr_{jk}}} \cdot N} \tag{26}$$

(It is important to note that these equations will reduce to equations (5) and (6) in the univariate two- facet example if each form is examined separately)

Thus, if the total available budget for the testing procedure in the salesperson's job performance measure is \$200 and the cost per occasion per rater is \$30, then the required number of occasions and raters, using the obtained variance and covariance components in Table 2, are $n_o = 2$, $n_r = 3$. The maximum generalizability coefficient would then be obtained by (11), and in this example provides the results presented in Table 3.

As can be seen in Table 3, the maximum generalizability coefficient is $\rho^2 = .86$ with .233, .061, .300 composite for the job performance measures. Webb et al. (1983) indicate that the analogy of factor analysis is helpful in understanding how to interpret the coefficients of the composites and the resulting dimensions. In summary, the coefficients ($\underset{\sim}{a}$) in the multivariate generalizability analysis presented in Table 3 are analogous to factor loadings, the dimensions are analogous to factors, and the composites are analogous to factor scores. Thus, the generalizability coefficient determined by the optimal solution under the imposed budget constraints indicates that the first dimension is a composite heavily weighted by the items on Form A and the items on Form C. It is important to note that similarly weighted composites were observed when the generalizability coefficient for an unconstrained problem was determined. For example, when $n_o = 1$ and $n_r = 1$ the coefficient was $\rho^2 = .62$ with a .113, .057, .299 composite. Therefore, imposing constraints on the generalizability analysis and then solving for the coefficients does not change the interpretation of the contribution of each form to the general composite.

5.2. The Three- Facet Multivariate Case

Consider a three- facet p x i x j x k multivariate design with ijk facets and with n_i, n_j, and n_k number of observations. For example, such a design might be used in a study of supermarket product sales strategies for three randomly selected products to examine the effects of price, display type, and time of advertising (Wilkinson, Maon, & Paksoy, 1982). In such a generalizability study, three products might be measured in terms of three price and display levels, on two occasions during the day at ten different supermarkets. If there is a limited budget available, we would need to find a set of values for the number of observations of facets that will maximize generalizability without violating the budget.

Unfortunately, the three- facet case is considerably more complicated than the two- facet case, and no closed form equations can be determined. Instead, by applying a similar approach to that used in the two-facet case, combined with the search procedure from the univariate three-facet problem, we will propose a simple search procedure that converges quickly to the optimal solution.

Similarly to the two- facet case, the function to be minimized is:

$$V_\delta = \frac{V_{pi}}{n_i} + \frac{V_{pj}}{n_j} + \frac{V_{pk}}{n_k} + \frac{V_{pij}}{n_i n_j} + \frac{V_{pik}}{n_i n_k} + \frac{V_{pjk}}{n_j n_k} + \frac{V_{pijk,\ e}}{n_i n_j n_k} \tag{27}$$

under the budgetary constraint that

$$c n_i n_j n_k \leq \bar{c} \tag{28}$$

As shown in the two-facet case, the simplest approach is to try and minimize all the elements δ_{rs} of the matrix V_δ together, namely:

$$\text{Min } \delta_{rs} = \frac{v_{pi\ rs}}{n_i} + \frac{v_{pj\ rs}}{n_j} + \frac{v_{pk\ rs}}{n_k} + \frac{v_{pij rs}}{n_i n_j} + \frac{v_{pik\ rs}}{n_i n_k} + \frac{v_{pjk\ rs}}{n_j n_k} + \frac{v_{pijk,\ e\ rs}}{n_i n_j n_k} \tag{29}$$

for r = 1, .., m and s = 1, .., n.

Once again, denote the solution of δ_{rs} by δ^*_{rs}, which basically becomes the goal of each δ_{jk} minimization. Any deviations from the goal can be obtained again by:

$$d_{rs} = \sigma_{rs} - \delta^*_{rs} \geq 0 \qquad \text{for all r and s.}$$

The minimization of the sum of deviations is:

$$\text{Min} \sum_k \sum_r d_{rs} \tag{30}$$

subject to (with some algebraic manipulation):

$$\frac{v_{pi\ rs}}{n_i} + \frac{v_{pj\ rs}}{n_j} + \frac{v_{pk\ rs}}{n_k} + \frac{v_{pij\ rs}}{N} \cdot n_k + \frac{v_{pik\ rs}}{N} \cdot n_j + \frac{v_{pjk\ rs}}{N} \cdot n_i - d_{rs} = \delta^*_{rs} \tag{31}$$

$$n_i n_j n_k = N \tag{32}$$

and

$$n_i, n_j, n_k \geq 0 \tag{33}$$

Using the Lagrange relaxation we get:

$$L(\lambda, n_i, n_j, n_k, d) = \sum_s \sum_r d_{rs} - \sum_s \sum_r \lambda_{rs} \left[\frac{v_{pi\ rs}}{n_i} + \frac{v_{pj\ rs}}{n_j} + \frac{v_{pk\ rs}}{n_k} + \right. \tag{34}$$

$$\left. \frac{v_{pij\ rs}}{N} \cdot n_k + \frac{v_{pik\ rs}}{N} \cdot n_j + \frac{v_{pjk\ rs}}{N} \cdot n_i - d_{rs} - \delta^*_{rs} \right]$$

$$- \lambda (n_i n_j n_k - N)$$

where $\lambda = \{ \lambda_{rs} \}$ for $r = 1, .., m$; $s = 1, .., n$

$d = \{ d_{rs} \}$ for $r = 1, .., m$; $s = 1, .., n$.

So by differentiation with respect to λ, λ_{rs}, d_{rs}, n_i, n_j, n_k and setting the results to zero we obtain at the optimal:

$$\alpha_i (n_i) = \alpha_j (n_j) = \alpha_k (n_k) \tag{35}$$

where

$$\alpha_x (n_x) = \sum_s \sum_r \left[\frac{v_{px\ rs}}{n_x} - \frac{v_{py\ rs}}{N} \cdot n_x \right]$$

with $x = i, j, k$
$y = jk, ik, ij$

and

$$n_i n_j n_k = N$$

From (35) we observe that at the optimal point of the solution there is a similar symmetric monotonically decreasing structure to the functions presented in the univariate three- facet case that can be used in the previously presented bisection procedure that converges quicly to the environment of the optimal solution. For this reason, we will only present an illustrative example.

6. Illustrative Example

The following example will illustrate how the proposed search procedure works for a three- facet multivariate case. Let us make use of the estimated variance and covariance components presented in Table 4. If the total budget available is \$600 and the cost per ijk (price, display, occasion) combination is \$50 (i.e. N= 12), using proposed procedure we can calculate the optimal number of pricing levels (n_i), display types (n_j), and occasions (n_k) to use within the specified budget that will give the largest coefficient of generalizability.

Table 5 presents the complete set of iterations in the search procedure, with the final number of observations after six iterations equal to $n_i = 2.75$, $n_j = 1.98$, and $n_k = 2.09$. If these values are rounded to the nearest integers we get the best possible solution close to the optimal with $n_i = 3$, $n_j = 2$, and $n_k = 2$. This combination of the number of observations of facets ijk yields the minimized sum of the deviations $\sum_s \sum_r d_{rs}$ which minimizes V_δ and maximizes ρ^2. Thus, in order to maximize generalizability in future D studies of supermarket product sales strategies under similar budget constraints, a researcher would need to study three pricing levels, two display levels, on two different measurement occasions. For example, a researcher might want to study on two different occasions regular, cost, and reduced prices along with normal and special types of displays. Table 6 presents the results of the multivariate generalizability analysis. Thus, using these values in equation (11), $\rho^2 = .70$ is the largest coefficient of generalizability that can be obtained without violating the budgetary constraint imposed.

7. Summary and Conclusions

In this paper we have presented a methodology for determining the optimal number of observations of facets to use in univariate and multivariate measurement designs that maximize generalizability when resource constraints are imposed. Using these sets of procedures, a decision maker can determine the number of observations that are needed to obtain the largest possible generalizability coefficient for a given amount of resources. Of course, if a decision maker wished to determine the minimum number of observations per subject for a specified generalizability coefficient, this could easily be obtained by using our procedure and a decision maker could then examine the tradeoff between the coefficient of generalizability and the total budget. Although the present paper only considered univariate and multivariate fully crossed two- and three- facet designs, parallel solutions can easily be obtained for other types of designs that might be encountered in practical marketing research applications.

Table 1

Estimated Variance - Components for G Study of Brand Loyaly*

Source of Variation		Estimated Variance Component
Persons	(p)	1.892
Items	(i)	0.389
Occasions	(o)	3.264
pi		0.083
po		0.235
io		0.025
Residual	(pro, e)	1.250

* Adopted from Rentz (1987)

Table 2

Estimated Variance- Covariance Components for Multivariate G Study of Job Performance

Source of Variation		Form A	Form B	Form C
Persons (p)	A	2.59	2.34	1.21
	B	2.34	6.37	2.71
	C	1.21	2.71	4.06
Occasions (o)	A	-0.05	-0.11	0.03
	B	-0.11	0.65	0.33
	C	0.03	0.33	0.23
Raters (r)	A	0.31	0.10	0.09
	B	0.10	0.86	0.01
	C	0.09	0.01	0.51
po	A	-0.11	-0.14	0.04
	B	-0.14	1.26	0.59
	C	0.04	0.59	0.37
pr	A	0.64	0.24	0.16
	B	0.24	1.11	0.08
	C	0.16	0.08	0.81
pro, e	A	2.63	0.94	0.02
	B	0.95	5.84	0.32
	C	0.02	0.32	1.62

Table 3

Generalizability and Canonical Coefficients for Decision Study

	Dimensions					
	Ia	Ib	IIa	IIb	IIIa	IIIb
Form A	.233	.113	-.400	-.303	-.449	-.315
Form B	.061	.057	-.142	-.117	.403	.266
Form C	.300	.299	.416	.270	-.123	-.090
Multivariate G	**.865**	**.621**	**.783**	**.422**	**.637**	**.291**

a. Constrained solution with n_o= 2 and n_r= 3.

b. Unconstrained solution with n_o= 1 and n_r= 1.

Table 4

Estimated Variance- Covariance Components for Multivariate G Study of Supermarkets

Sourse of Variation		Product A	Product B	Product C
p	A	0.70	0.64	0.88
	B	0.64	0.65	0.71
	C	0.88	0.71	0.98
pi	A	0.18	0.32	0.09
	B	0.32	0.13	0.08
	C	0.09	0.09	0.33
pj	A	0.13	0.08	0.15
	B	0.08	0.10	0.07
	C	0.15	0.07	0.10
pk	A	0.12	0.11	0.09
	B	0.11	0.13	0.02
	C	0.09	0.02	0.17
pij	A	0.07	0.03	0.01
	B	0.03	0.10	0:02
	C	0.01	0.02	0.05
pik	A	0.12	0.07	0.06
	B	0.07	0.10	0.03
	C	0.06	0.03	0.11
pjk	A	0.03	0.02	0.01
	B	0.02	0.05	0.08
	C	0.01	0.08	0.06
pijk, e	A	0.63	0.91	0.16
	B	0.91	1.57	0.21
	C	0.16	0.21	1.26

Note: The variance- covariance components for the I, J, K, IJ, IK, JK, and IJK sources of variations are not displayed in the table because they are not needed for solving the optimization model presented.

Table 5

Search Procedure Iterations

Iteration	n_i	n_j	n_k	$n_i n_j n_k$	upper n_i	lower n_i	$\frac{\text{upper + lower}}{2}$
1	1	0.75	0.71	0.53	18.70	1	**9.85**
2	9.85	5.92	9.43	552.28	9.85	1*	**5.41**
3	5.41	3.60	4.70	91.95	5.42	1*	**3.21**
4	3.21	2.28	2.49	18.22	3.21	1.76	**2.49**
5	2.49	1.79	1.85	8.25	3.01	2.48	**2.75**
6	2.75	1.98	2.98	11.38	2.75	2.42	**2.59**

* Indicates value rounded to 1.

Table 6

Generalizability and Canonical Coefficient for Decision Study of Supermarkets

Product	Dimension		
	I	II	III
A	.715	-1.699	-5.802
B	-.226	2.168	3.109
C	.388	-0.101	2.618
Multivariate G	.703	.361	0a

a. Value rounded to zero.

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